



## **FLEXIBLE EXCHANGE RATE AND GROWTH OF A SMALL OPEN MONETARY ECONOMY WITH IMPORTED GOOD AND EXTERNALITIES**

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### **Abstract**

*This paper proposes a growth model of a small open economy with economic structure, flexible exchange rate, money policy and imported good. The model integrates the basic features of the Solow growth model, Uzawa's two-sector model, the monetary growth model with the MIU approach, and the neoclassical growth model for a small open economy. We simulate the model and demonstrate that the system has a unique saddle equilibrium point. We follow transitional behavior of the economy over a short-run period of time. We examine effects of changes in the money policy, the propensity to consume import good, the propensity to hold real money, the price of the imported good, the externalities of the industrial sector, the propensity to save. The comparative dynamic analysis provides some important insights. For instance, we show that as the money policy is increased, both the real money held by the household and the value of physical wealth is increased. This conclusion is consistent with the conclusion on the relation between money and physical wealth by Tobin (1956). As our model is more comprehensive than the Tobin model, it explains some phenomena which the Tobin model cannot explain. In our model as the monetary policy is increased, the output of the service sector and the sector's two inputs are increased, while the output of the industrial sector and the sector's two inputs are reduced; the trade balance is improved; the capital intensities of the two sectors and wage rate are reduced; the consumption levels of the industrial good, the service, and the imported goods are all increased; the inflation rate is reduced and the domestic currency is more appreciated. The inflation policy benefits the national economy as well as the household.*

**Keywords:** Exchange Rate, Capital Accumulation, Imported Good, Economic Structure, Small Open Economy

**JEL classification:** F11, F31

### **1. Introduction**

This study is focused on the impact of monetary policy on economic variables, such as prices, exchange rates, production and consumption in an open small economy. As monetary policy in a small open



economy with a flexible exchange rate may affect the gap between savings and investment, the policy can affect economic growth. As observed by Shaw et al. (2005: 720), “Though monetary policy is important in an open economy, until now very few efforts have been made to analyze the role of monetary policy for an open economy in the endogenous growth literature.” There are a few growth models of small open economies in which monetary policies are taken into account. For instance, Palokangas (1997) treats money as an intermediary good that reduces transaction costs. The model examines the impact of a change in the money policy on the long-run growth rate. Obstfeld (1981a, 1981b) analyzes the impact of monetary policies on a small Ramsey-growth economy with the rate of time preference as an increasing function of instantaneous utility. The models by Obstfeld introduce money into the neoclassical growth theory with the money-in-utility (MIU) approach by Sidrauski (1967). This study also introduces money with the money-in-utility approach, but with an alternative utility function proposed by Zhang (1993). This study is focused on the role of money on a small open monetary economy with endogenous capital accumulation and structural changes.

It is well-known that the seminal contribution in the field of monetary economic growth theory was made by Tobin (1956), who deals with an isolated economy in which outside money competes with real capital in the portfolios of agents. Tobin generalizes the Solow model by introducing money into the neoclassical growth theory. This study includes money into the growth model with the money in the MIU function approach. In this approach money is held as money yields some services. The approach includes money by entering real balances directly into the utility function.<sup>1</sup> The MIU approach was initially proposed by Patinkin (1965), Sidrauski (1967) and Friedman (1969). It should be noted that the article by Sidrauski (1967) challenges Tobin’s non-neutrality result. Sidrauski shows that money is superneutral in steady state and changes in the inflation rate have no effect on all the real variables in the economy.<sup>2</sup> Later on, it has been demonstrated that Sidrauski’s conclusions are dependent on the specific set-up of the model. For instance, the superneutrality in Sidrauski’s model is no longer true if variables such as leisure are included in the utility function. According to Wang and Yip (1992), the non-superneutrality is closely related to the signs of the cross-partial derivatives of the utility function with respect to consumption, leisure, and real balances.<sup>3</sup> The model with endogenous investment by Meng and Yip (2004) shows that physical capital is significant for stabilizing the real side of the economy when the monetary authority follows interest-rate feedback rules.<sup>4</sup> This study is strongly influenced by the traditional monetary growth models.

As small open economies are easily affected by disturbances such as global economic crisis, terms of trade and prices of input factors, many studies have been carried out to examine different exogenous changes on small open economies.<sup>5</sup> For instance, economists show a great interest in the impact of a

<sup>1</sup> See Eden (2005: Chap. 2) for the reasons why money is introduced into the utility function.

<sup>2</sup> Superneutrality of money means that the growth rate of money has no effect on the real equilibrium.

<sup>3</sup> It should be noted that Feenstra (1986) shows that the MIU approach is functionally equivalent to the transactions-cost model. It has also been shown that either the shopping-time model or cash-in-advance model can be rewritten as a MIU model.

<sup>4</sup> There are many other issues and models related to interactions between monetary policy and economic growth, for instance, Gomme (1993); van der Ploeg and Alogoskoufets (1994); Jones and Manuelli (1995); Dotsey and Starte (2000); Chappell and Matthews (2001); Chang et al. (2007); Nelson (2008) and Handa (2009).

<sup>5</sup> For instance, Sachs (1982); Svensson and Razin (1983); and Matsuyama (1987).



change in a country's terms of trade on the economy's growth.<sup>6</sup> This study deals with effects of changes in international economic conditions and domestic preference changes on trade balance, exchange rate and economic structural change. To deal with the interactions among growth, economic structure, exchange rate and trade balance, we need a genuine dynamic framework. Nevertheless, there are only a few dynamic economic models which address these interactions in an integrated framework. This study deals with exchange rate and growth by developing a monetary economic growth model with economic structural change and imported goods within an integrated analytical framework.

This paper examines the response of exchange rate, inflation rate, economic growth and trade balance in a small open two-sector economy with capital accumulation to changes in the price of imported goods, the propensity to save, and preference for imported goods. The introduction of money to growth theory is influenced by Chao et al. (2006) and Zhang (2013). The description of the real aspects is framed with the Uzawa two-sector growth model. A main different between our approach and the traditional models is that this study is based on an alternative utility function proposed by Zhang (1993). The rest paper is organized as follows. Section 2 defines the basic model. Section 3 shows how we solve the dynamics and simulates the model. Section 4 examines effects of changes in some parameters on the economic system over time. Section 5 concludes the study. The appendix proves the main results in Section 3.

## 2. The monetary growth model with imported goods

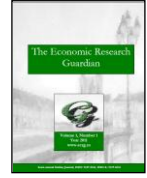
The model is a combination of the basic features of a few well-known economic models. These models are the Solow growth model, the two-sector growth model of Uzawa (1961), the monetary growth model with the MIU approach, and the neoclassical growth model for a small open economy. The small open economy produces two goods: an internationally traded good (called industrial good) and a non-traded good (called services).<sup>7</sup> To examine effects of terms of trade and foreign trade, as in Eicher *et al.* (2008), we include imported goods which is not produced by the economy, but consumed by the domestic consumers. The price of the industrial good is unity. We assume that the economy is too small to affect the world interest rate  $r^*$  and price of imported good  $p_Z$ . Both  $r^*$  and  $p_Z$  are assumed constant during the study period. The household holds wealth and receives income from wages, and interest payments of wealth. All markets are perfectly competitive and capital and labor are completely mobile between the two sectors. Capital is perfectly mobile in international market. There is no international emigration or/and immigration. Labor is homogeneous and is fixed. We use subscript index,  $i$  and  $s$ , to denote respectively the industrial and service sectors.

The economy produces services and traded good. The price of traded good is given in the world market. We assume that there is no impediment to trade. Capital is perfectly mobile domestically as well as

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<sup>6</sup> For instance, Mendosa (1995); Kose (2002); and Turnovsky and Chattopadhyay (2003).

<sup>7</sup> According to Brock (1988) goods and services are classified as traded and non-traded. Brock studies the dynamic adjustment of the relative price of non-traded and the current account as government purchases, changes in tax income or investment subsidy are exogenously changed. This study focuses the effects of changes in the the propensities to save and to consume imported goods and terms of trade.



internationally. Capital depreciates at a fixed rate,  $\delta_k$ , which is independent of the manner of use. Domestic and foreign assets are perfect substitutes. We use  $P(t)$ ,  $\hat{P}(t)$ , and  $e(t)$  to represent respectively the domestic price of the traded good, the foreign price level (normalized to one later on) and the nominal exchange rate. The purchasing power parity holds

$$e(t)\hat{P}(t) = P(t). \quad (1)$$

The uncovered interest parity also holds

$$r(t) = r^*, \quad (2)$$

where  $r(t)$  is the real interest rate on domestic assets. From (1) we also have  $\pi(t) = \hat{\pi}(t) + \varepsilon(t)$ , where  $\pi(t) \equiv \dot{P}(t)/P(t)$  is the rate of inflation of the traded good in terms of the domestic currency,  $\hat{\pi}(t)$  is the rate of inflation of the traded good in terms of foreign currency assumed to be exogenously given to the small open economy, and  $\varepsilon(t) \equiv \dot{e}(t)/e(t)$  is the rate of depreciation of the domestic currency. As  $\hat{\pi}(t) = 0$ , we have

$$\pi(t) = \varepsilon(t). \quad (3)$$

### Industrial sector

We use  $K_j(t)$  and  $N_j(t)$  to stand for the capital stocks and labor force employed by sector  $j$ ,  $j = i, s$ , at time  $t$ . The function  $F_j(t)$  is the output level of sector  $j$ . Following Zhang (2009), we use the following production function

$$F_i(t) = \Omega_i(t) K_i^{\alpha_i}(t) N_i^{\beta_i}(t), \quad \alpha_i, \beta_i > 0, \quad \alpha_i + \beta_i = 1, \quad (4)$$

where  $\alpha_i$  and  $\beta_i$  are parameters and  $\Omega_i(t)$  is a term measuring externalities. We assume that the capital stock is closely related to the scale of production and the scale yields externalities. The impact of externalities of the industrial sector on the total productivity  $\Omega_i(t)$  is specified  $\Omega_i(t) = A_i K_i^{\theta_i}(t)$ ,  $A_i, \theta_i \geq 0$ , where  $K_i^{\theta_i}(t)$  the effect of externalities. When  $\theta_i = 0$ , there is no externality.

Markets are competitive; thus labor and capital earn their marginal products, and firms earn zero profits. The real wage rate,  $w(t)$ , is determined in domestic labor market. The marginal conditions for the industrial sector are

$$r_\delta = \alpha_i \Omega_i(t) k_i^{-\beta_i}(t), \quad w(t) = \beta_i \Omega_i(t) k_i^{\alpha_i}(t), \quad (5)$$

where  $k_i(t) \equiv K_i(t)/N_i(t)$  and  $r_\delta \equiv r + \delta_k$ .



### Service sector

The production function of the service sector is

$$F_s(t) = \Omega_s(t) K_s^{\alpha_s}(t) N_s^{\beta_s}(t), \quad \alpha_s, \beta_s > 0, \quad \alpha_s + \beta_s = 1, \quad (6)$$

where  $\alpha_s$  and  $\beta_s$  are parameters and  $\Omega_s(t)$  is the impact of externalities on the service total productivity. We specify  $\Omega_s(t)$  as follows  $\Omega_s(t) = A_s K_s^{\theta_s}(t)$ ,  $A_s, \theta_s \geq 0$ .

We use  $P_s(t)$  to stand for the nominal price of the service. The marginal conditions for the service sector are

$$r_s = \alpha_s \Omega_s(t) p(t) k_s^{\alpha_s-1}(t), \quad w(t) = \beta_s \Omega_s(t) p(t) k_s^{\alpha_s}(t), \quad (7)$$

where  $k_s(t) \equiv K_s(t)/N_s(t)$  and  $p(t) \equiv P_s(t)/e(t)$ .

### The inflation policy

This study introduces money by following the traditional monetary growth theory. Money is introduced by assuming the existence of a central bank. The role of the central bank is to distribute at no cost to the population a per capita amount of fiat money  $M(t) > 0$ . The scheme according to which the money stock evolves over time is deterministic and known to all agents. With the inflation policy  $\mu$  being the constant net growth rate of the money stock,  $M(t)$  evolves over time according to  $\dot{M}(t) = \mu M(t)$ ,  $\mu > 0$ .

The government expenditure in real terms per capita,  $\tau(t)$ , is given by

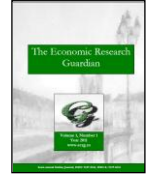
$$\tau(t) = \frac{\mu M(t)}{e(t)} = \mu m(t), \quad (8)$$

where  $m(t) \equiv M(t)/e(t)$ . The representative household receives  $\mu m(t)$  units of paper money from the government through a “helicopter drop”, also considered to be independent of his money holdings. From  $M(t) = m(t)e(t)$ , we have

$$\varepsilon(t) = \frac{\dot{e}(t)}{e(t)} = \mu - \frac{\dot{m}(t)}{m(t)}. \quad (9)$$

### Behavior of domestic households

This study models consumers’ behavior by the approach proposed by Zhang (1993). The implications of this approach are similar to those in the Keynesian consumption function and models based on the



permanent income hypothesis, which are empirically much more valid than the approaches in the Solow model or the in Ramsey model. We use  $\bar{k}(t)$  to stand for the real value of physical wealth per household. The household's current income from the interest payment,  $r\bar{k}(t)$ , the wage payments,  $w(t)$ , and the distributed money from government,  $\tau(t)$ , is given by  $y(t) = r\bar{k}(t) + w(t) - \pi(t)m(t) + \tau(t)$ , where  $\pi(t)m(t)$  is the cost of holding money. If we neglect the possibility that the household borrows for current spending and assume that the household can sell and buy the value of physical wealth  $\bar{k}(t)$  instantaneously without any transaction cost, then the total value of disposable wealth,  $a(t)$ , to sell for purchasing goods and services and for saving is the sum the money held and the value of physical wealth, i.e.,  $a(t) = \bar{k}(t) + m(t)$ . The disposable income of a household is defined as the sum of the current income and the value of disposable wealth, i.e.,  $\hat{y}(t) = y(t) + a(t)$ . That is

$$\hat{y}(t) = a(t) + r\bar{k}(t) + w(t) - \pi(t)m(t) + \tau(t). \quad (10)$$

The disposable income is used for saving and consumption. At time  $t$  the consumer has the total amount of income equaling  $\hat{y}(t)$  to distribute among saving, holding money, and consuming.

As in Zhang (2013), we assume that the household's utility is dependent on the value of money held  $m(t)$ , the consumption levels of service  $c_s(t)$ , industrial good  $c_i(t)$ , and the imported good  $c_z(t)$ , and saving  $s(t)$ . The utility function is specified as follows  $U(t) = \theta m^{\varepsilon_0}(t) c_s^{\gamma_0}(t) c_i^{\xi_0}(t) c_z^{\varsigma_0}(t) s^{\lambda_0}(t)$ ,  $\varepsilon_0, \gamma_0, \xi_0, \varsigma_0, \lambda_0 > 0$ , in which  $\varepsilon_0, \gamma_0, \xi_0, \varsigma_0$  and  $\lambda_0$  are a typical household's elasticity of utility with regard to money, service, industrial good, imported good, and saving. We call  $\varepsilon_0, \gamma_0, \xi_0, \varsigma_0$ , and  $\lambda_0$  the propensities to hold money, to consume service, to consume industrial good, to consume imported good, and to hold wealth, respectively.

At each point of time, the household distributes the disposable income among holding money, consuming goods and services, and saving. The budget constraint is

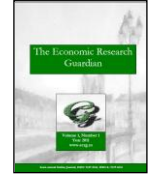
$$(1 + r)m(t) + p(t)c_s(t) + c_i(t) + p_z c_z(t) + s(t) = \hat{y}(t). \quad (11)$$

Insert (10) in (11)

$$\bar{\pi}(t)m(t) + p(t)c_s(t) + c_i(t) + p_z c_z(t) + s(t) = \bar{y}(t) \equiv (1 + r)\bar{k}(t) + w(t) + \tau(t), \quad (12)$$

where  $\bar{\pi}(t) \equiv \pi(t) + r^*$ , which is the opportunity cost of holding money. Maximizing  $U(t)$  subject to (6) yields

$$\bar{\pi}(t)m(t) = \bar{\varepsilon} \bar{y}(t), \quad c_s(t) = \frac{\gamma \bar{y}(t)}{p(t)}, \quad c_i(t) = \xi \bar{y}(t), \quad c_z(t) = \varsigma \bar{y}(t), \quad s(t) = \lambda \bar{y}(t), \quad (13)$$



where  $\bar{\varepsilon} \equiv \rho \varepsilon_0$ ,  $\gamma \equiv \rho \gamma_0$ ,  $\xi \equiv \rho \xi_0$ ,  $\varsigma \equiv \frac{\rho \varsigma_0}{p_z}$ ,  $\lambda \equiv \rho \lambda_0$ ,  $\rho \equiv \frac{1}{\varepsilon_0 + \gamma_0 + \xi_0 + \varsigma_0 + \lambda_0}$ .

The real wealth changes as follows

$$\dot{a}(t) = s(t) - a(t). \quad (14)$$

This equation simply says that the change in the wealth is equal to the saving minus the dissaving.

### Demand of and supply for services

The equilibrium condition for services is

$$c_s(t)N = F_s(t). \quad (15)$$

### Full employment of capital and labor

The total capital stocks employed by the country,  $K(t)$ , is employed by the two sectors. The full employment of labor and capital is represented by

$$K_i(t) + K_s(t) = K(t), \quad N_i(t) + N_s(t) = N. \quad (16)$$

### Trade balance

Let  $\bar{K}(t)$  stand for the total asset owned by the country's population, that is,  $\bar{K}(t) = \bar{k}(t)N$ . We use  $E(t)$  to denote the balance of trade. We have

$$E(t) = r^* (\bar{K}(t) - K(t)). \quad (17)$$

We have thus built the dynamic growth model with inflation and exchange rate.

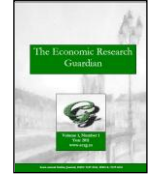
## 3. The Dynamics of the National Economy

The appendix shows that the motion of the economic system is determined by two differential equations. The following lemma shows how we can determine the motion of all the variables in the dynamic system.

### Lemma 1

The motion of  $k_i(t)$  and  $m(t)$  is determine by

$$\dot{k}_i(t) = \bar{\Lambda}(k_i(t), m(t)), \quad \dot{m}(t) = \tilde{\Lambda}(k_i(t), m(t)), \quad (18)$$



in which  $\bar{\Lambda}(k_i, m)$  and  $\tilde{\Lambda}(k_i, m)$  are functions of  $k_i(t)$  and  $m(t)$  defined in the appendix. We determine all the other variables as functions of  $k_i(t)$  and  $m(t)$ :  $\bar{k}(t)$  by (A10)  $\rightarrow k_s(t)$  by (A2)  $\rightarrow w(t)$  by (A1)  $\rightarrow K_i(t)$  by (A9)  $\rightarrow K_s(t)$  by (A7)  $\rightarrow N_j(t) = K_j(t)/k_j(t)$ ,  $j = i, s \rightarrow \Omega_i(t)$  and  $\Omega_s(t)$  by the specified forms  $\rightarrow F_j(t)$  by the specified forms  $\rightarrow p(t)$  by (A3)  $\rightarrow \bar{y}(t)$  by (12)  $\rightarrow \pi(t)$  by (13)  $\rightarrow \varepsilon(t) = \pi(t) \rightarrow e(t)$  by (9)  $\rightarrow P(t)$  by (1)  $\rightarrow c_s(t)$ ,  $c_i(t)$ ,  $c_z(t)$ , and  $s(t)$  by (13)  $\rightarrow K(t) = K_i(t) + K_s(t) \rightarrow \bar{K}(t) = \bar{k}(t)N \rightarrow E(t)$  by (17).

From Lemma 1 we determine the motion of economic system at any point of time. If we determine the motion of the two variables by the two differential equations, we determine the motion of the whole system. It should be mentioned that as demonstrated Turnovsky (1996) in a model of a small open economy in which domestic capital accumulation involves convex costs of adjustment, the equilibrium growth rates of domestic capital and consumption are determined largely independent. The former is determined by production conditions,<sup>8</sup> the latter is determined primarily by tastes. In our model, these variables are closely related to each other. The domestic capital and consumption are determined jointly by production conditions and preference as demonstrated late on.

As the expressions are too tedious, it is difficult to get explicit conclusions. For interpretation, we simulate the model. We specify parameter values as follows

$$\begin{aligned} r^* &= 0.03, \quad p_z = 1, \quad \delta_k = 0.05, \quad N = 10, \quad \mu = 0.02, \quad A_i = 1.3, \quad A_s = 0.9, \\ \alpha_i &= 0.3, \quad \alpha_s = 0.34, \quad \theta_i = 0.07, \quad \theta_s = 0.06, \quad \lambda_0 = 0.6, \quad \xi_0 = 0.15, \quad \gamma_0 = 0.06, \\ \eta_0 &= 0.06, \quad \zeta_0 = 0.02, \quad \varepsilon_0 = 0.01. \end{aligned} \quad (19)$$

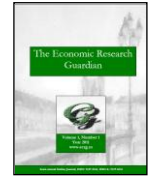
The rate of interest is fixed at 3 per cent and the price of imported good is one. The depreciation rate is 5 per cent. The population is 10. The propensity to save is 0.6. The propensity to consume is 0.15, which is much higher than the propensity to consume services, or propensity to consume imported goods. It should be remarked that although the specified values are not based on empirical observations, the choice does not seem to be unrealistic. For instance, some empirical studies<sup>9</sup> on the US economy demonstrate that the value of the parameter,  $\alpha$ , in the Cobb-Douglas production is approximately equal to 0.3.

To simulate the model, we specify the following initial condition  $k_i(t) = 13$ ,  $m(t) = 2$ . We plot the motion of the dynamic system in Figure 1. We plot the motion only for 4 periods. As shown below, the system has a saddle point. In the long term the economic system may not converge to its equilibrium. The output level of the industrial sector rises, while the output level of the service rises over the simulation period. The economy experiences deflation. The trade is in surplus until it turns to be in defect near the end of the simulation period. The capital wealth owned by the household falls, while the

<sup>8</sup> This conclusion is obtained in some other models of small open economies with perfect capital mobility and perfect substitutability between home capital and foreign bonds (see, for instance, Zeira, 1987).

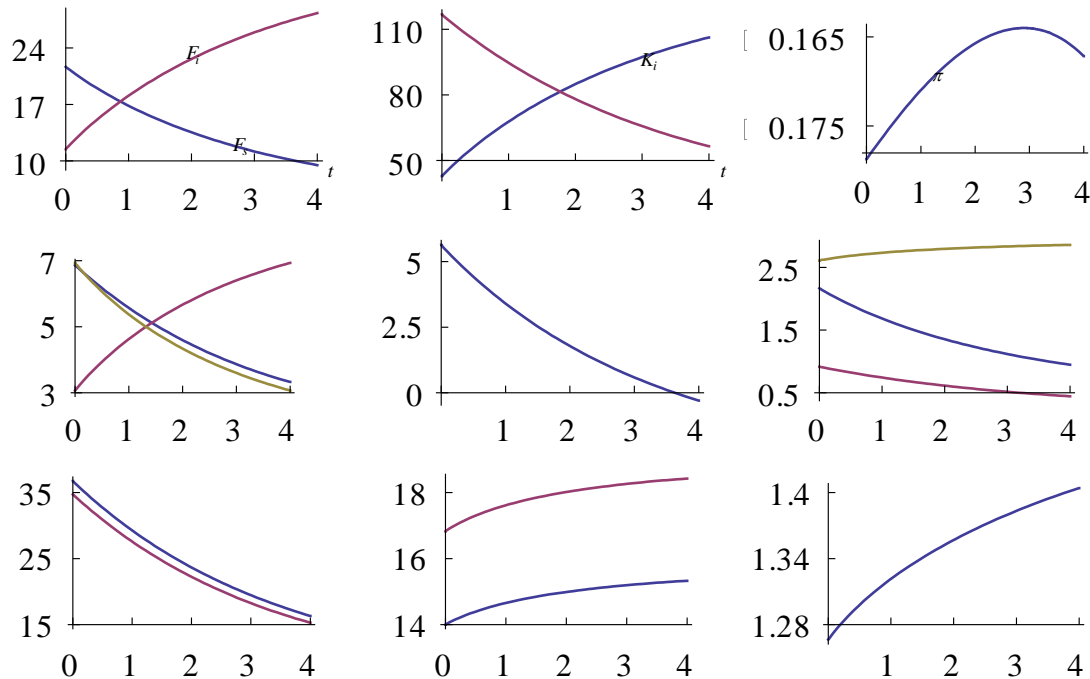
<sup>9</sup> For instance, Miles and Scott (2005); and Abel, et al. (2007).





real money changes slightly. The consumption levels of the goods and services are reduced. The labor force is shifted from the service sector to the industrial sector. The capital intensities and wage rate rise.

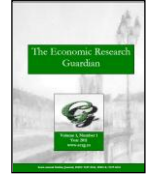
Figure 1 - The Motion of the National Economy



It is straightforward to demonstrate that the economic system has a unique equilibrium point. We calculate the equilibrium values of the variables as follows

$$w = 2.95, \quad p = 1.54, \quad \bar{k} = 2.78, \quad m = 1.39, \quad a = 4.17, \quad N_i = 9.07, \quad N_s = 0.93, \quad K_i = 143.16, \\ K_s = 17.73, \quad F_i = 38.18, \quad F_s = 2.72, \quad E = -4.00, \quad k_i = 15.79, \quad k_s = 18.98, \\ c_i = 1.04, \quad c_s = 0.27, \quad c_z = 0.14.$$

The eigenvalues at the equilibrium point are -0.28, 0.09. Accordingly, the unique equilibrium point is unstable. In general situations, the system will not converge to its equilibrium point. As our study shows how to determine the dynamic path for any initial conditions, we can follow the motion of the system over time.



#### 4. Comparative Dynamic Analysis

The previous section shows how to follow the motion of all the variables of the dynamic system. This section carries out comparative dynamic analysis with regard to some parameters. As we have shown how to simulate the motion of the system, it is straightforward to make the analysis.

##### **A rise in the inflation policy**

First, we examine the impact of the following change in the money policy:  $\mu_0: 0.02 \Rightarrow 0.03$ . We introduce a variable,  $\bar{\Delta}x(t)$ , to stand for the change rate of the variable,  $x(t)$ , in percentage due to changes in the parameter value. It should be noted that for trade balance we use  $\Delta x(t)$  to stand for change. As the government prints more money, both the real money held by the household and the value of physical wealth is increased. According to Tobin (1956), the main merit of Tobin's model is that the model extends the Keynesian reasoning to a situation of the long run. Tobin (1956: 684) states: "In classical theory, the interest rate and the capital intensity of the economy are determined by 'productivity and thrift', that is, by the interaction of technology and saving propensities ... Keynes gave reasons why in the short run monetary factors and portfolio decisions modify, and in some circumstances dominate, the determination of the interest rate and the process of capital accumulation. I have tried to show here that a similar proposition is true for the long run."

This study extends Tobin's basic ideas on money and growth to a monetary growth economy with economic structure. It is well known that in the Tobin model an increase in the level of the inflation policy will increase the capital stock of an economy. Our model predicts the same effect as the Tobin model does with regard to the national capital stock.

Our model explains economic structural change in association with a rise in the monetary policy which the Tobin model cannot explain. There are economic structural changes in association with the change in the monetary policy. The output of the service sector and the sector's two inputs are increased, while the output of the industrial sector and the sector's two inputs are reduced. The trade balance is improved. The capital intensities and wage rate are reduced. The consumption levels of the goods and service are all increased. It can be seen that the rise in the inflation policy benefits the national economy as well as the household. It should be noted that our conclusion is similar to what Braumann (2000) observes that the factor of open economy plays a significant role in the negative relationship between inflation and economic growth (see also Gylfason and Herbertsson, 2001).

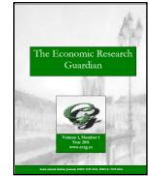
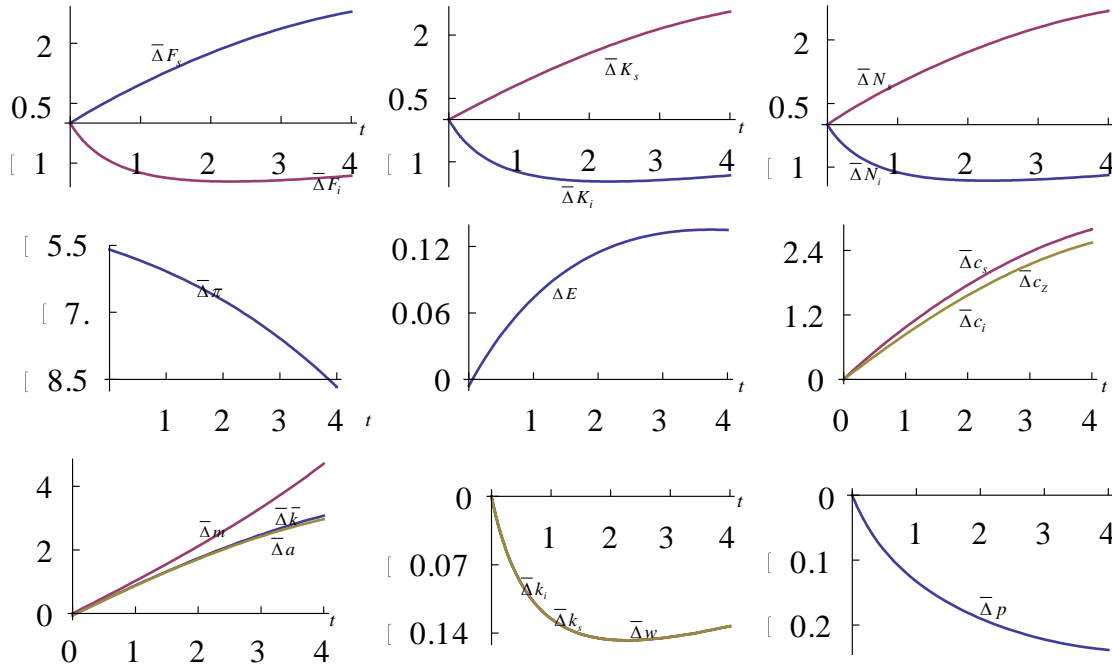


Figure 2 - A Rise in the inflation policy



### A fall in the propensity to hold money

We examine the impact of the following change in the propensity to hold money:  $\varepsilon_0: 0.01 \Rightarrow 0.008$ . As the propensity to hold money is reduced, the real money is increased. This happens because the inflation rate falls greatly. The total wealth and the value of physical wealth are reduced. In association with the fall in the disposable income, the consumption levels of the industrial good, imported good and service are reduced.

The price of service rises. The output level of the service sector and the sector's two inputs are reduced. The output level of the industrial sector and the sector's two inputs are increased. The capital intensities and wage rate are increased.

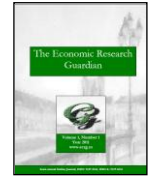
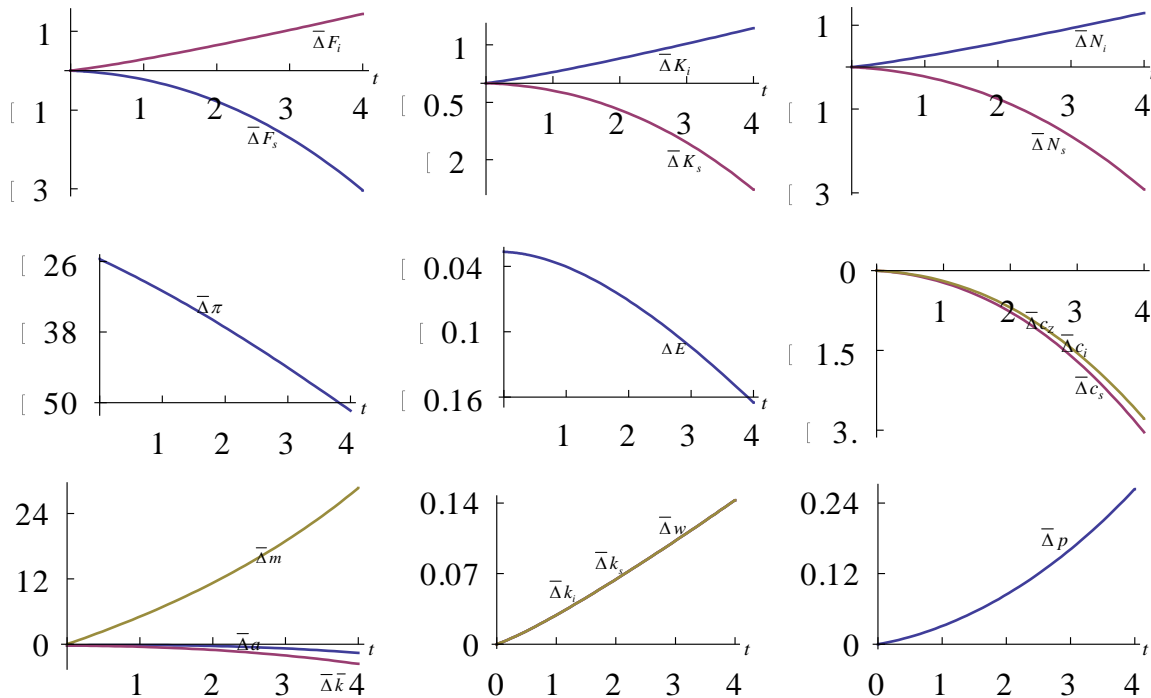


Figure 3 - A fall in the propensity to hold money



### A rise in the propensity to consume imported good

We examine the impact of the following change in the propensity to consume imported goods:  $\zeta_0: 0.02 \Rightarrow 0.03$ . The increased demand for the imported good results in the increase in consumption of the good with the condition that the price is fixed in the international market. More consumption on the imported good is associated with the falling consumptions of the industrial good and service. In association with falling in the demand for services, the output level of the service sector and the two inputs of the sector are reduced. The labor is shifted to the industrial sector.

The industrial sector increases its labor employment and production scale. The inflation rate falls. The trade balance is improved initially and deteriorated in later on. The value of physical wealth is increased initially but is reduced late on. The real money held by the household is initially slightly affected but is increased late on. The capital intensities and wage rate are increased.

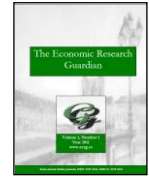
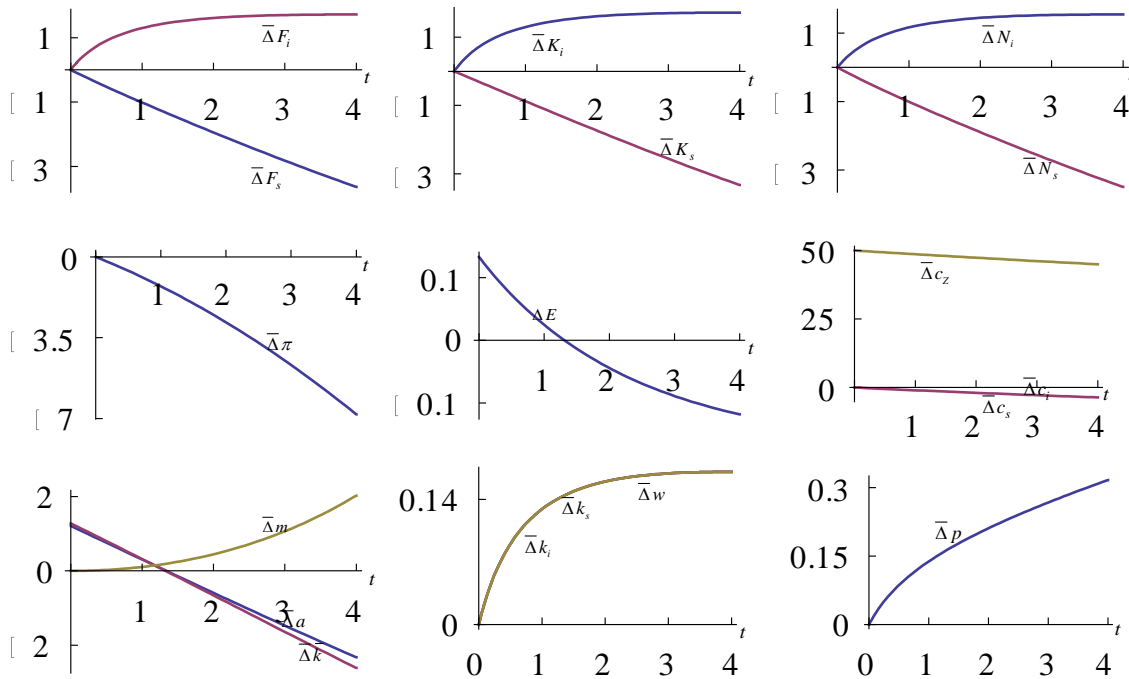


Figure 4 - A rise in the propensity to consume the imported good



### Externalities strengthened

We now study the case that the externalities of the industrial sector are strengthened as follows:  $\theta_i: 0.07 \Rightarrow 0.075$ . The capital intensities and wage rate are increased as the externalities are increased. As the externalities of industrial sector are increased and the externalities of the service are not changed (when the sector's capital is not changed) and the capital intensities are changed in the same rate, the labor force is shifted from the industrial sector to the service sector. Initially the industrial sector's output and two inputs are reduced; but near the end of the simulation period the industrial sector and capital input are slightly increased and the labor input is still reduced. The service sector's output level and two inputs are increased.

The total wealth and the value of physical wealth are increased. In association in rising inflation rate, the real money is reduced. The trade balance is improved. The consumption levels of the goods and service are increased. The price of the service falls initially but rises late.

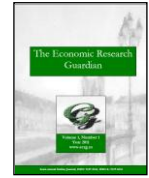
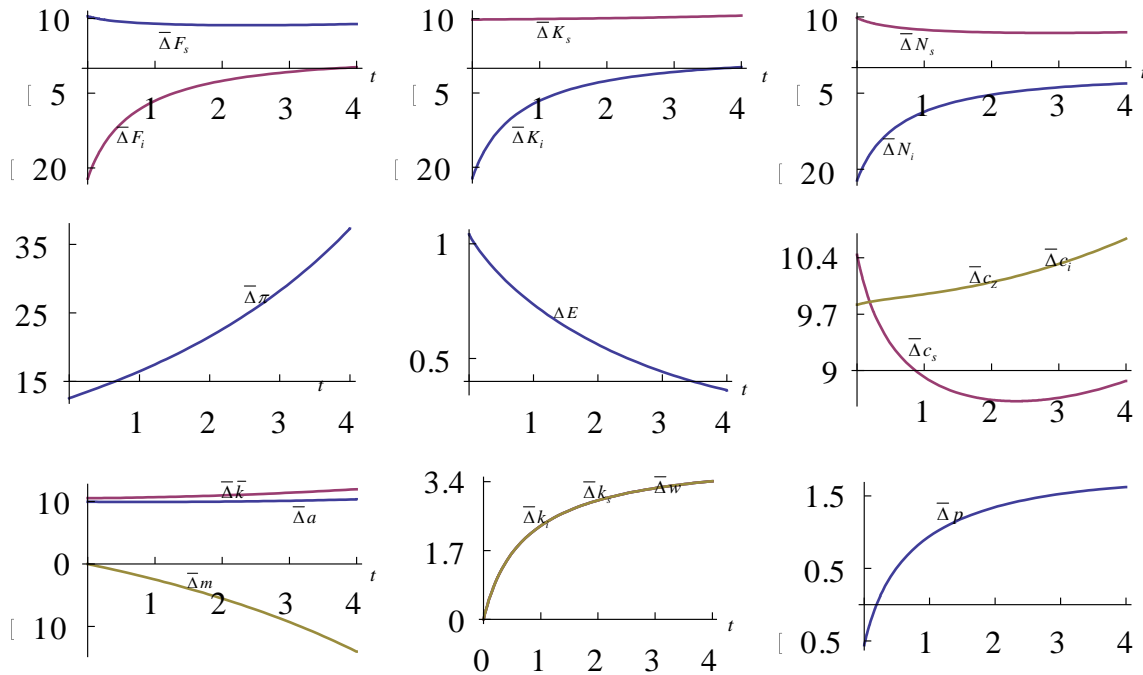


Figure 5 - The externalities of the industrial sector are strengthened



### A rise in the propensity to save

It is well known that Adam Smith and Keynes give the opposite answers to the effects of a change in the propensity to save. Only a few theoretical models address issues related to effects of changes in the propensity to save on monetary growth economies. We now examine how changes in the preference affect the national economy. We specify the preference change as follows:  $\lambda_0: 0.6 \Rightarrow 0.65$ . The rise in the propensity to save increases the value of physical wealth and the total wealth but slightly reduces the real money. The trade balance is improved. In association with the rise in the disposable income, the consumption levels of the industrial good, service and imported goods are increased.

The wage rate, the capital intensities and the price of the service are reduced slightly. As the demand for service is increased, the output level of the service sector and the sector's two inputs are increased. In association of the shift of the labor force from the industrial sector to the service sector, the output level of the industrial sector and the sector's two inputs are reduced.

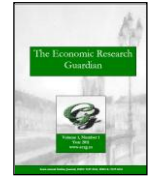
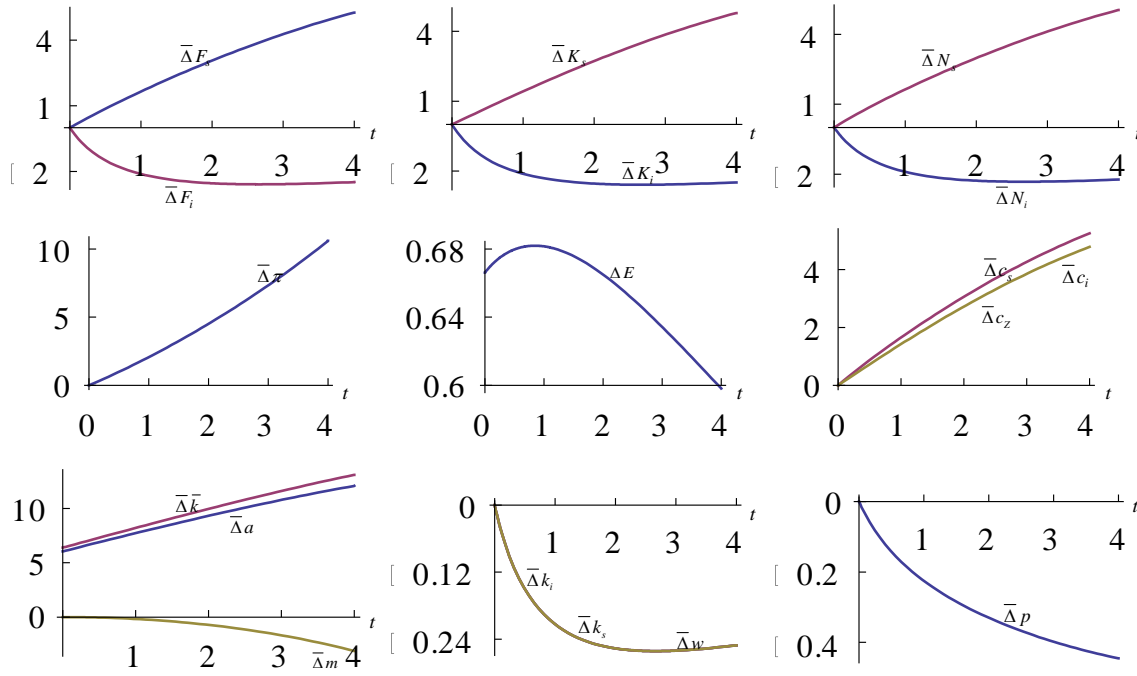
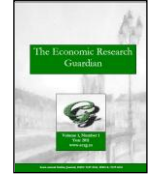


Figure 6 - A rise in the propensity to save



## 5. Conclusions

This paper proposed a growth model for a small open economy with economic structure, flexible exchange rate, money policy and imported goods in a perfectly competitive economy. The model integrated the basic features of a few well-known economic models in a single framework. These models include the Solow growth model, Uzawa's two-sector growth model, the monetary growth model with the MIU approach, and the neoclassical growth model for a small open economy. The study focuses on effects of changes in terms of the money policy, the propensity to save, the rate of interest rate, the price and preference for imported goods on the dynamic paths of trade balance, exchange rate and economic growth. In this model, the production side is the same as in the neoclassical growth theory with externalities. We used Zhang's utility function to describe behavior of the household with the real money, saving, and the consumption levels of the industrial good, service and imported good as the variables that directly affect the utility. We simulated the model and demonstrated that the system has a unique saddle equilibrium point. We were concerned with transitional behavior of the economy over a short-run period of time. We examined effects of changes in the money policy, the propensity to consume import good, the propensity to hold real money, the price of the imported good, the externalities of the industrial sector, the propensity to save. The comparative dynamic analysis provides some important insights. For instance, as the money policy is increased, both the real money held by the household and the values of physical wealth are increased. This conclusion is consistent with the conclusion on the



relation between money and physical wealth by Tobin (1956). As our model is more comprehensive than the Tobin model, it explains some phenomena which the Tobin model cannot explain. In our model as the monetary policy is increased, the output of the service sector and the sector's two inputs are increased, while the output of the industrial sector and the sector's two inputs are reduced; the trade balance is improved; the capital intensities of the two sectors and wage rate are reduced; the consumption levels of the industrial good, the service, and the imported goods are all increased; the inflation rate is reduced, and the domestic currency is more appreciated. It can be seen that the rise in the inflation policy benefits the national economy as well as the household. Finally, it should be remarked that there are different ways in introducing money into economic analysis. Except the MIU approach accepted in this study, money is assumed to affect productivity.<sup>10</sup> The domestic output is produced by combining physical capital and real money as follows:  $F(k, m)$ . In this approach, firms hold money to facilitate production on the grounds that money enables them to economize the use of inputs. We may generalize our approach by integrating this approach in a more general analytical framework.

## Acknowledgements

The author is grateful to constructive comments of Editor Mihai Mutascu and the anonymous referee.

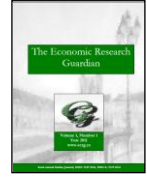
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<sup>10</sup> For instance, Levhari and Patinkin (1968); Friedman (1969), and Fischer (1979).





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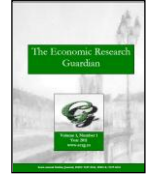
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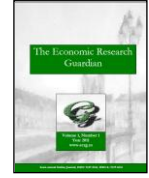
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## Appendix

### Proving Lemma

From (5), we get

$$\Omega_i = \frac{r_\delta k_i^{\beta_i}}{\alpha_i}, \quad w = r_w k_i, \quad (\text{A1})$$

where  $r_w \equiv \beta_i r_\delta / \alpha_i$ . From (5) and (4), we have

$$k_s = \bar{\alpha} k_i, \quad (\text{A2})$$

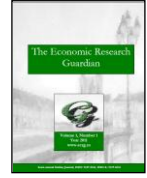
where  $\bar{\alpha} = \alpha_s \beta_i / \beta_s \alpha_i$ . From (4), we solve

$$p = \frac{r_\delta k_s^{\beta_s}}{\alpha_s \Omega_s}. \quad (\text{A3})$$

Substitute  $c_s = \gamma \bar{y} / p$  into (14)

$$\gamma \bar{y} N = p F_s. \quad (\text{A4})$$

Insert  $r_\delta = \alpha_s p F_s / K_s$  in (A4)



$$\bar{y} = n_0 K_s, \quad (A5)$$

where  $n_0 \equiv r_\delta / \alpha_s \gamma N$ . Substitute the definition of  $\bar{y}$  into (A5)

$$\tilde{r} \bar{k} + r_w k_i + \tau = n_0 K_s, \quad (A6)$$

where we use  $w = r_w k_i$  and  $\tilde{r} \equiv 1 + r$ . Insert  $k_j = K_j / N_j$  in (16)  $\frac{K_i}{k_i} + \frac{K_s}{k_s} = N$ .

Substituting (A4) and (A2) into the above equation yields

$$K_i + \frac{K_s}{\bar{\alpha}} = N k_i. \quad (A7)$$

Insert (A6) in (A7)

$$K_i = \left( \frac{N n_0 \bar{\alpha} - r_w}{\bar{\alpha} n_0} \right) k_i - \frac{\tilde{r} \bar{k} + \tau}{\bar{\alpha} n_0}. \quad (A8)$$

From (A1) and the definition of  $\Omega_i$ , we have

$$K_i = \left( \frac{r_\delta}{A_i \alpha_i} \right)^{1/\theta_i} k_i^{\beta_i/\theta_i}. \quad (A9)$$

Substitute (A9) into (A8)

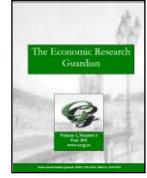
$$\bar{k} = \Lambda(k_i, m) \equiv \left( \frac{N n_0 \bar{\alpha} - r_w}{\tilde{r}} \right) k_i - \left( \frac{r_\delta}{A_i \alpha_i} \right)^{1/\theta_i} \frac{\bar{\alpha} n_0}{\tilde{r}} k_i^{\beta_i/\theta_i} - \frac{\mu}{\tilde{r}} m, \quad (A10)$$

where we also use  $\tau = \mu m$ . We determine  $\bar{k}$  as a function of  $k_i$  and  $m$ . From  $\bar{\pi} m = \bar{\varepsilon} \bar{y}$  in (13) and  $\pi = \varepsilon$ , we have

$$r m = \bar{\varepsilon} ((1 + r)\Lambda + r_w k_i + \mu m) - \varepsilon m, \quad (A11)$$

where we use (A10) and the definition of  $\bar{y}$ . Insert (9) in (A11)

$$\dot{m} = \tilde{\Lambda}(k_i, m) \equiv (r + \mu - \mu \bar{\varepsilon})m - \bar{\varepsilon} ((1 + r)\Lambda + r_w k_i). \quad (A12)$$



By the following procedure we can determine all the variables as functions of  $k_i(t)$  and  $m(t)$ :  $\bar{k}$  by (A10)  $\rightarrow k_s$  by (A2)  $\rightarrow w$  by (A1)  $\rightarrow K_i$  by (A9)  $\rightarrow K_s$  by (A7)  $\rightarrow N_j = K_j/k_j$ ,  $j = i, s \rightarrow \Omega_i$  and  $\Omega_s$  by the specified forms  $\rightarrow F_j$  by the specified forms  $\rightarrow p$  by (A3)  $\rightarrow \bar{y}$  by (12)  $\rightarrow \pi$  by (13)  $\rightarrow \varepsilon = \pi \rightarrow e$  by (9)  $\rightarrow P$  by (1)  $\rightarrow c_s, c_i, c_z$  and  $s$  by (13)  $\rightarrow K = K_i + K_s \rightarrow \bar{K} = \bar{k} N \rightarrow E$  by (17). By (14) and this procedure, we have

$$\dot{\bar{k}} = \Omega(k_i, m) \equiv \lambda \bar{y} - \Lambda - m - \tilde{\Lambda}, \quad (\text{A13})$$

where we use  $a = \bar{k} + m$  and (A11). As the expression is tedious, we will not present it explicitly. Taking derivatives of (A10) with respect to time yields

$$\dot{\bar{k}} = \frac{\partial \Lambda}{\partial k_i} \dot{k}_i + \frac{\partial \Lambda}{\partial m} \tilde{\Lambda}, \quad (\text{A14})$$

where we also use (A12). Solve (A13) and (A14)

$$\dot{k}_i = \bar{\Lambda}(k_i, m) \equiv \left( \Omega - \frac{\partial \Lambda}{\partial m} \tilde{\Lambda} \right) \left( \frac{\partial \Lambda}{\partial k_i} \right)^{-1}. \quad (\text{A15})$$

From (A12) and (A15), we determine the motion of  $k_i(t)$  and  $m(t)$ . The values of all the other variables are determined by the procedure described before. We thus proved Lemma 1.