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HOW A PAY-AS-YOU-GO PENSION SYSTEM CAN LEAD TO A PARETO IMPROVEMENT IN AN OLG MODEL WITH ENDOGENOUS FERTILITY

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Abstract
Mostly, all developed countries have the problem that the total fertility rate is below its sustainable level. Therefore, all these countries face economic problems caused by a demographic change. In this paper, we show that given certain conditions a pay-as-you-go pension system where the pension depends only on the number of own children leads to a Pareto improvement and to an increase of the total fertility rate. To show this we use an Overlapping Generation model with endogenous fertility in a small open economy.

Keywords: Endogenous fertility, Pay-as-you-go pension system, OLG model

JEL classification: H55, J13

1. Introduction

We observe in mostly all developed countries that the total fertility rates are below their sustainable level, which would be theoretically two children per female on average. According to the CIA Factbook (2012), the total fertility rate is in 94 countries below two children per female. Surprisingly, the lowest fertility rates are observed in developed Asian countries, Macau (0.92 children/female), Hong Kong (1.07), Singapore (1.11), Taiwan (1.15), Japan (1.21) and South Korea (1.23), also in all European countries, except France the total fertility rate is below its sustainable level.[1] The problem is that, a decreasing population leads to a demographic change, which increases the ratio between the number of retired persons to the number of workers. Additionally, in the majority of all developed countries, a kind of PAYG pension system is established. The problem caused by a declining population is, how to avoid a decline of the pension and at the same time how to avoid an unacceptable increase of the contribution rate. The usual way out of policy-makers is to introduce a kind of an additional fully-funded pension system. The huge disadvantage of this policy is, that at

[1] For example Romania (1.31), Germany (1.42), Austria (1.42), Italy (1.41), Greece (1.4), Poland (1.32).

minimum one generation has to bear a double burden, its members have to contribute to both systems, where the internal interest rate of the PAYG pension for this generation will become zero or negative. Besides this problem, the demographic change also raises the costs of the health care system, because the health care expenditures for old people are higher than for young people. However, the easiest way to solve all these problems is to increase the fertility rate. The idea here is to introduce a pension system which gives parents an economic incentive to increase the number of children.

This kind of pension system is not new, in principle, it is an old idea, because the system existed implicitly at the times before public pension systems were introduced. The pension payment should depend only on the number of own children. The problem of the usual existing pay-as-you-go (PAYG) pension system is that it creates a free-rider problem with respect to the fertility behavior. Every young person expects that the others will get enough children so that the sustainability of the PAYG pension system is guaranteed. The result is the fertility rate declined so dramatically that the sustainability of the whole economy is taken into question.

In general, we know from Verbon (1988) that the introduction of a Pay-as-you-go (PAYG) pension system leads to a welfare reduction, if the fertility rate is lower than the interest rate, if the fertility rate is exogenous and if no market distortions exist as for example in Homburg (1990). In this paper, we remove the assumption that the fertility rate is exogenously given. This idea is not new, and was earlier done, for example, by Galor & Weil (1996), Wigger (1999), Fanti & Gori (2008), Stauvermann (1996), Srinivasan (1995), van Groezen et al. (2003), Kolmar (1997) and Fenge & Meier (2005, 2009) and others. Here we follow explicit the model of van Groezen et al. (2003), and take the idea of a child factor in a PAYG pension system of Kolmar (1997) and Fenge & Meier (2005, 2009) into account, even that they focus on other subjects than the one in this paper. I.e. Kolmar (1997) focuses on the comparison of different PAYG pension systems and a fully funded pension system, while Fenge & Meyer (2005, 2009) analyzes the differences and similarities between a specific PAYG pension system and child allowances. In their model, the amount of the individual pension depends partly on the number of own children. We go a step further to the extreme and assume that the individual pension depends totally on the number of children. For example, if someone has no children, she will get no pension.[2] In such a world and considering a small open economy with perfect international capital markets, it can be shown that an expansion or introduction of a PAYG pension system will increase the welfare under certain conditions. Especially we will show that a transition from usual income-dependent PAYG pension to a PAYG pension system can be realized without harming any generation.

In the next section, we introduce the model and derive the market equilibrium. In section 3, we will analyze the equilibrium, and we will show that the introduction or a transition to such a child-depending PAYG pension system leads to a Pareto improvement. Additionally, we will show that the fertility rate increases, if the child-dependent PAYG pension system is introduced or extended. In the last section, we summarize and discuss the results.

[2] Of course, we have to assume that everyone can get children.

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2. The model

We assume a small open economy with a perfect international capital market. There is only one good in the economy, which can be consumed or invested. The production side of the economy should be represented by standard neoclassical production function without exogenous growth like in Diamond (1965). Because of the assumption of a small open economy, the wage rate *w* and interest factor *R* of this economy given by the international capital market. The structure of the model goes back to Samuelson (1958) and Diamond (1965). The main difference to their models is that we allow endogenous fertility decisions made by the individuals. Therefore, in each period, two generations make decisions, a young generation, which offers labor inelastically and decides on the number of children; and an old generation, which lives only from its savings and pension.

The utility of a representative individual is given by the following log-linear utility function, which is commonly used in OLG-models with endogenous fertility.;[3]

$$U_t = \ln c_t^1 + q\ln c_{t+1}^2 + v\ln (N_t), \qquad (1)$$

The utility U_t depends on the consumption in the first period of life c_t^1, the consumption in the second period of life c_{t+1}^2, and the number of children $N_t = 1 + n_t$. The parameter *q* represents the individual discount factor, where $0 < q \leq 1$. The positive parameter *v* reflects the preference to get descendants. The innovation of this model is that we analyze a PAYG pension system with child factor, where the child factor equals one. The idea of a child factor goes back to Fenge & Meier (2005), where they assumed that the child factor is smaller than one. That means that the pension of an individual depends only on the number of own children. In some sense, this pension system is like a intra-family pension system in former times, but with the difference that the government determines the pension payment and the contribution rate. By doing this the moral hazard problem which can emerge in an intra-family pension system is non-existent. Then the pension payment is given by:

$$p_{t+1} = dN_{t+1}. \qquad (2)$$

Here p_{t+1} represents the pension in period *t*+1 of an individual born in period *t*. The variable $d \geq 0$ stands for a constant pension contribution rate, which every individual has pay in its first period of life and N_{t+1} the number of children of the individual. The individual is confronted with the following budget constraints.

$$c_t^1 = w_t - bN_{t+1} - d - s_t \qquad (3)$$

and

[3] See for example Galor & Weil (1996), Fanti& Gori (2008), van Groezen et al. (2003), Wigger (1999) and many others.

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$$c_{t+1}^2 = R_{t+1}s_t + dN_{t+1}. \quad (4)$$

It should be noted that the savings can be negative, because for an individual there are no capital market restrictions in our model. In the first period of life the individual earns w_t and contributes d to the pension system, and $b > 0$ are the costs to rear a child and N_{t+1} is the number of children. In the second period of life it receives the amount d times the number of children N_{t+1} as its pension and the savings s_t times the interest factor R_{t+1}. If the savings are negative, the debt including interest factor must be paid back to the lender, which is of course no problem because of the pension. Now we formulate the maximization of the representative individual, by using (1)-(3):

$$L\left(c_t^1, c_{t+1}^2, N_{t+1}, \mu\right) = \ln c_t^1 + q\ln c_{t+1}^2 + v\ln\left(N_{t+1}\right) - \mu\left(c_t^1 + \frac{c_{t+1}^2 - dN_{t+1}}{R_{t+1}} + bN_{t+1} + d - w_t\right) \quad (5)$$

At next, we determine the first order conditions:

$$\frac{1}{c_t^1} - \mu = 0, \quad (6)$$

$$\frac{q}{c_{t+1}^2} - \frac{\mu}{R_{t+1}} = 0, \quad (7)$$

$$\frac{v}{N_{t+1}} - \mu\left(b - \frac{d}{R_{t+1}}\right) = 0 \quad (8)$$

$$c_t^1 + \frac{c_{t+1}^2 - dN_{t+1}}{R_{t+1}} + bN_{t+1} + d - w_t = 0. \quad (9)$$

From (6) and (7) we get:

$$c_{t+1}^2 = qR_{t+1}c_t^1, \quad (10)$$

and from (6) and (8):

$$N_t = \frac{vc_t^1}{bw_t}. \quad (11)$$

Substituting equations (10) and (11) in (9); and after doing simple reformulations, we get for the consumption in the first period of life:

$$c_t^1 = \frac{w_t - d}{(1+q+v)}. \quad (12)$$

The consumption in the second period of life is given by:

$$c_{t+1}^2 = \frac{qR_{t+1}(w_t - d)}{(1+q+v)}. \quad (13)$$

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We get for the fertility factor N_t or number of children:

$$N_t = \frac{\gamma R_{t+1}(w_t - d)}{(1+q+v)(bR_{t+1} - d)}. \qquad (14)$$

3. The Analysis of the Model

Now we have determined the values of all relevant variables in a steady-state equilibrium. From (14) we can derive, that an interior solution is only guaranteed, if $b > \frac{d}{R_{t+1}}$ holds.[4] This condition means that the discounted pension, which a parent can expect per child, is smaller than the costs to rear the child. Now we can examine the equilibrium values.

Proposition 1: If the contribution rate *d* increases and if $w_t > bR_{t+1}$, the number of children will also increase.
Proof:
We differentiate (14) with respect to the contribution rate *d* and it results:

$$\frac{\partial N_t}{\partial d} = \frac{\gamma R_{t+1}(w_t - bR_{t+1})}{(1+q+v)(bR_{t+1} - d)^2} > 0, \text{ if } w_t > bR_{t+1}. \qquad (15)$$

The assumption $w_t > bR_{t+1}$ requires that the discounted gross income of an individual exceeds the costs to rear it. It can be assumed that this assumption is fulfilled in reality.
At next, we will substitute the equilibrium values into the utility function (1), where we omit the time indices, because all variables are time independent in the steady-state:

$$U(d) = \ln\left(\frac{w-d}{(1+q+v)}\right) + q\ln\left(\frac{qR(w-d)}{(1+q+v)}\right) + v\ln\left(\frac{vR(w-d)}{(1+q+v)(bR-d)}\right) \qquad (16)$$

If we analyze (16) and its characteristics with respect to the contribution rate *d,* then we should only take into account reasonable values of *d*. This leads immediately to the restriction, that $w > d$ must hold. So we can restrict the further analysis to $d \epsilon [0, w[$.

Proposition 2: It exists always a pension contribution rate $\bar{d} < bR$ in this PAYG pension system, so that $U(\bar{d}) > U(0)$, if *w*>*bR*. Therefore, a Pareto-improvement can be realized by the introduction of this PAYG pension system.

[4] If $b \leq \frac{d}{R_{t+1}}$ would be allowed, the number of children would reach its biological maximum, or infinite if reproductive medicine would able to realize it.

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It should be noted, that our concept of welfare takes only individuals into account, who are actually born. So we avoid a discussion like in Golosov et al (2007) about the welfare of unborn individuals.

Proof:
At first we differentiate the utility function of a working individual in period *t*. We get for the first derivative of the utility function (16) with respect to *d*,

$$\frac{\partial U}{\partial d} = \frac{v(w-bR)}{(w-d)(bR-d)} - \frac{(1+q)}{(w-d)} \lessgtr 0 . \qquad (17)$$

If $\frac{v(w-bR)}{(bR-d)} > 1+q$ holds, the derivative is positive. Now we determine the limit value of $\frac{v(w-bR)}{(bR-d)}$, where *d* strives toward the value *bR:*

$$\lim_{d \to bR} \frac{v(w-bR)}{(bR-d)} = \infty \qquad (18)$$

This means if *d* strives toward *b*R, the numerator is positive and the denominator is striving toward zero. Hence, the expression (18) strives toward infinity. Thus, the marginal utility (17) is then positive. In a next step, we have to show that the level of utility, if *d* strives toward *b*R, exceeds utility, if $d = 0$. Because of that we determine the limit value of the utility function:

$$\lim_{d \to bR} \ln\left(\frac{w-d}{(1+q+v)}\right) + q\ln\left(\frac{qR(w-d)}{(1+q+v)}\right) + v\ln\left(\frac{vR(w-d)}{(1+q+v)(bR-d)}\right) = \infty \qquad (19)$$

Please note, (19) describes the utility of an individual, which has to contribute to the pension system and receives a pension. The first two summands are constant, and the third summand strives toward infinity, because the denominator of the third summand is striving toward zero. That means, it exists always a contribution rate $\bar{d} < bR$ so that $U(\bar{d}) > U(0)$, because the value of $U(0)$ is always a well defined number. If the pension system is introduced in period *t*, then the old generation is obviously better off, because it did not contribute anything to the pension system, but it receives a pension payment. Alternatively and more importantly, if a usual PAYG pension system existed, the old generation will still receive its pension payments and its utility remains unchanged. This makes a transition from an usual income-dependent PAYG pension system to a child-dependent PAYG pension system is possible, and that without any transition phase.

The proposition tells us, that for a small open economy, with perfect international capital markets exists always a PAYG pension system where the pension depends on the number of own children so that the utility of all generations can be increased, as long as *w*>*b*R. Additionally, the results show that it is possible to make a PAYG pension reform, without harming any generation. The intuition behind the result is, in the introduction phase of the proposed child dependent PAYG pension system the working generation still contributes to the existing PAYG pension system, which was not dependent on the number of children. Then the old generation is able to receive its pensions. That means a double burden, which has to be borne by one generation, induced by an additional fully-

funded pension system can be avoided. Therefore, the introduction of an additional fully-funded pension system, like it was done in Germany or Sweden, was not necessary.
Of course, this result is based only on a theoretical consideration, where we implicitly assumed that the human reproduction rate is in principle unlimited.[5]

The intuition behind this result is, even that the investment in human reproduction is less profitable than an investment in physical capital, because of $b > \frac{d}{R}$, the additional utility generated by additional children offsets the implicit interest rate difference between investments in physical capital and in human reproduction. Of course, we do not propose to increase the number of children more than its sustainable level, because too many children cause in general also strong economic and more important ecological problems, which are beyond the scope of this simple model.

4. Conclusions

In this paper, we show that a PAYG pension scheme exists that is superior to a capital funded pension system. [6] Additionally, we have shown that a Pareto-improving transition from a conventional PAYG pension system, which is irrespective of the number of children, to a PAYG pension system, where the pension is dependent on the number of children is possible. Furthermore, we have shown that this PAYG pension system can increase the number of children to its sustainable level and the welfare of the economy. Of course, this is only a theoretical result, and in this simple form, the PAYG pension system should not be applied in reality, because some people are unable to get children because of different serious reasons. Nevertheless, the model indicates the direction in which a PAYG pension system should be developed to increase the total fertility rate of an economy to avoid the problems of a demographic change.
However, instead of our proposed PAYG pension system, the introduction of child allowances in an economy with a usual PAYG pension system would lead to the same results, because both policy measures are equivalent as it was shown implicitly by van Groezen et al. (2003) and Fenge & Meyer (2005, 2009). Alternatively, debt-financed child allowances would also generate the same results as our PAYG pension system, as long as the taxes to finance the government debt are lump sum taxes.

However, it should be noted again that the general idea is not new; this kind of pension system was implicitly voluntarily established long before public pension systems were introduced. It eroded as individuals were compelled to become more mobile and more independent from their families. The result was a kind of free-rider behavior, which can be avoided by a public PAYG pension system like it is proposed in this paper.

[5] If we would assume that the human reproduction rate is limited by a certain number N^{max}, then we will realize sometimes a corner solution, which leads to an OLG model, which is very similar to the standard OLG model, except that the number of children generates utility and to raise them is not costless. However it easy to show that, if in such a model $N^{max} > R$ holds, the results of our model will not change with respect to the utility. In general, this assumption is fulfilled.
[6] It is well known that private savings and a capital funded pension system are perfect substitutes.

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