CORRUPTION, NATIONAL DEBTS, EDUCATION AND GROWTH

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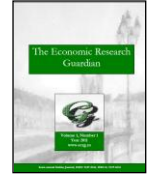
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Abstract

The main purpose of this study is to analyze dynamic relations between debts and corruption. We develop a dynamic general equilibrium model with endogenous wealth, human capital, corruption, and government debt. The model is a synthesis of Solow's growth model, Uzawa's two-sector model, Diamond's government debt model, and Uzawa-Lucas' human capital growth model with Zhang's approach to household behavior. We construct and simulate the model. The simulated case has two equilibrium points, the one with negative government debt being stable and the other one with positive debt being unstable. We carry out comparative dynamic analysis separately with regard to the two equilibrium points to see transitory and long-run effects of changes in some parameters. We also point out limitations and possible extensions in the conclusion.

Keywords: Corruption rates, Government debt, Economic growth, Education, Human capital, Time distribution

JEL classification: O41, D3, E2
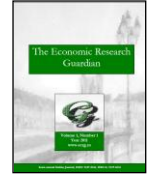
1. Introduction

Corruption is a global problem (Lui, 1996). It occurs in different parts or processes of social and economic life. Causes and consequences of corruption are complicated and not easy to identify. It is obvious that characters, scales and scope of corruption are related to stages of economic growth. Some theoretical models on corruption and economic growth are published (e.g., Becker, 1968; Rose-Ackerman, 1999; Shi and Temzelides, 2004; Dzhumashev, 2014; D'Agostino, *et.al.* 2016). Many empirical studies on corruption and economic growth are conducted (e.g., Mauro, 1995, 1998; Ehrlich and Lui, 1999; Del Monte and Papagni, 2001; Rivera-Batiz, 2001; Glaeser and Saks, 2006; Swaleheen and Stansel, 2007; Teles, 2007; Gyimah-Brempong, 2002; Chea, 2015). The literature on corruption is too vast to be properly reviewed here. Different researchers emphasize different causes, channels and consequences of corruption. Like with regard any important issue in social sciences, neither general convergent theoretical nor convergent empirical conclusion on the role of corruption on economic growth is obtained. Chea (2015: 187) reveals "From the theoretical background and empirical evidence

from various studies spread in various countries in the world using different methods over the years, there are mostly negative findings of economic growth. Nevertheless, there are also positive findings found on the effects of corruption on economic growth." Leff (1964) revealed possible positive effects of corruption on economic development. Long time ago Huntington (1968) pointed out that the "efficient corruption" in the form of bribery may enable firms to get works completed in a political economic environment plagued by bureaucratic complexity. Corruption may reduce operations costs so that the efficiency of the whole system is improved (Mo, 2001). Myrdal (1968) pointed out that corruption brings about economic loss as officials tend delay permissions of projects in order to get bribes. The recent review by Dzhumashev (2014: 203) concludes that "the existing literature lacks a more general approach for interpreting the role of governance, the size government, and the level of development in the relationship between corruption and growth." Although different researches provide many insights into complicated reasons, processes and consequences of corruption, it might be said that there only a few theoretical dynamic models which deal with issues related to interdependence between growth, corruption, wealth accumulation, and human capital dynamics. The purpose of this study is to make a contribution to the literature on growth and corruption by designing a general dynamic equilibrium framework to reveal consequences of different corruptions. This is a new contribution to the literature in the sense that it deals with the complicated issues within a general equilibrium framework.

Government debts are important issues faced by many governments in the contemporary world. Debts belong to these kinds of economic problems which have to be dealt with in a dynamic general equilibrium framework. Debt is accumulated and its change is related to related to economic growth, tax income, interest payments of debt, government fiscal policies, and government expenditures. As shown Zhang (2005), theoretical economics still lacks a proper and effective analytical framework for examining complicated dynamic problems. Zhang (2005) tries to change the situation by suggesting an alternative approach to behavior of household to the traditional Solow or Ramsey approaches. This study applies Zhang's approach to examine dynamics of debts with corruption. Fiscal policies are emphasized in some theoretical models as determinants of persistent economic growth (Hochman, 1981; Wijkander, 1984; Barro, 1990; Turnovsky, 2000, 2004; Glomm and Ravikumar, 1997; Gómez, 2008; and Park, 2009). We assume that the government is responsible for providing public services with a single input, officials. The model in this study also includes another important issue of modern economies. Heavy government debts are characterized of many contemporary economies. It is often observed that growth models with endogenous debts may have locally unique or indeterminate equilibrium point, depending on whether the government uses income taxes, consumption taxes or other policies (e.g., Judd, 1987; Schmitt-Grohe and Uribe, 1997; and Mankiw and Weinzierl, 2006). Our model also identifies multiple equilibria. As this study provides a computational procedure to follow motion of the system with any initial conditions, we can easily show how each equilibrium point is affected in short term as well as in long term with transitory processes explicitly plotted when some exogenous conditions are changed.

This paper makes a unique contribution to the literature of economic theory with regard to interdependence between growth, wealth accumulation, human capital change, government debt, corruption, and government taxation by synthesizing Solow's growth model, Uzawa's two-sector model, Diamond's government debt model, and Uzawa-Lucas' human capital growth model with Zhang's approach to household behavior. Modelling of economic structure and production is much influenced by neoclassical growth theory (Solow, 1956; Burmeister and Dobell, 1970; Zhang, 2005). The

interdependence between education and growth follows the Uzawa-Lucas two-sector model (Uzawa, 1965; Lucas, 1988). There are many generalizations and extensions of the standard Uzawa-Lucas model (e.g., Jones *et al.* 1993; Stokey and Rebelo, 1995; Mino, 1996, 2001; Zhang, 2003; Alonso-Carrera and Freire-Seren, 2004; De Hek, 2005; Chakraborty and Gupta, 2009; and Sano and Tomoda, 2010). We follow this tradition, even though we analyze household behavior with Zhang's approach (Zhang, 1993, 2005). It should be mentioned that this model is to integrate the model of endogenous human capital with education sector by Zhang (2016) with the model of trade with national debts by Zhang (2015) and the model of endogenous corruption by Zhang (2017). The organization of the rest paper is as follows. In Section 2 we develop the basic model with dynamic interactions between endogenous wealth, human capital, corruption, and government debts within a general dynamic equilibrium framework. In Section 3 we provide a computational program which makes it easy to follow the motion of the dynamic system with computer and simulate the model. In Section 4 we carry out comparative dynamic analysis to show effects of changes in some parameters on the economic system over time. In Section 5 we conclude the study. In the Appendix we prove the results in Section 3.

2. The basic model

The model in this study is to integrate the model with endogenous human capital with education sector by Zhang (2016) with the model with endogenous corruption by Zhang (2017). The economy is composed of one industrial sector, one public sector, and one education sector. The industrial sector produces a single industrial good in the economy. We choose the price of the industrial good to be unit and to measure all prices. The industrial sector is basically similar to the sector in the standard one-sector neoclassical growth model (Solow, 1956; Burmeister and Dobell, 1970; Barro and Sala-i-Martin, 1995). We classify the population into two homogeneous groups with fixed numbers, officials and workers. Workers are fully employed by the industrial and education sectors. The officials are fully employed by the public sector. Work hours of each group are endogenous. Assets of the economy are owned by officials and workers. Wealth accumulation and human capital are endogenous The education and industrial sectors use labor and capital inputs and public services to produce. All markets are perfectly competitive. Factors are fully utilized at every moment. We introduce the following variables

j - subscripts for workers ($j = 1$) and officials ($j = 2$), respectively;

$\bar{N}_j$ and $\bar{N}$ - fixed population of group j and the total population, $\bar{N} = \bar{N}_1 + \bar{N}_2$;

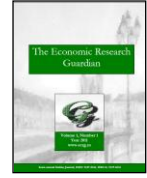$r(t)$ and $w_j(t)$ - rate of interest and group $j's$ wage rate per unit of time;

$p(t)$ - price of education per unit time;

$T_j(t)$, $\bar{T}_j(t)$ and $\tilde{T}_j(t)$ - work, leisure, and education time of group $j's$ household;

$c_j(t)$ and $s_j(t)$ - consumption and saving of group $j's$ household;

$N_j(t)$ and $h_j(t)$ - labor supply and human capital of group j;

$N_q(t)$ and $K_q(t)$ - group 1'labor force and capital employed by sector q, $q = i, e$;

$K(t)$ and $D(t)$ - total capital of the economy and the government's debt;

$d_j(t)$ - the amount of the government's debt owned by group $j's$ representative household;

$\bar{k}_j(t)$ and $a_j(t)$ - value of physical wealth and total value of wealth owned by group $j's$ representative household;

τ_q - the government's tax rate on sector $q's$ output, $q = i, e;$;

ϕ_q and $\bar{\phi}_q$ - fixed corruption rate on output level of sector q and $\bar{\phi}_q \equiv 1 - \phi_q - \tau_q$;

ϕ_a and $\bar{\phi}_a$ - corruption rate on workers' income from wealth and $\bar{\phi}_a \equiv 1 - \phi_a$;

ϕ_w and $\bar{\phi}_w$ - corruption rate on workers' wage income and $\bar{\phi}_w \equiv 1 - \phi_w$;

δ_k and δ_{hj} - depreciation rates of physical capital and group $j's$ human capital.

We have $N_j(t)$ as follows

$$N_j(t) = h_j^{m_j}(t)\bar{N}_j T_j(t), \tag{1}$$

where m_j is group $j's$ utilization efficiency of human capital. As only workers are engaged in education sector and workers are fully employed, we have

$$N_i(t) + N_e(t) = N_1(t). \tag{2}$$

The industrial sector

We specify the industrial sector's production function as follows

$$F_i(t) = A_i G^{\theta_i}(t) K_i^{\alpha_i}(t) N_i^{\beta_i}(t), \quad \alpha_i + \beta_i = 1, \quad A_i, \theta_i, \alpha_i, \beta_i > 0, \tag{3}$$

where A_i, θ_i, α_i and β_i are parameters. Different from Chen and Guo (2013) but similar to de Vaal and Ebben (2011), we take account of possible effects of public goods $G(t)$ on productivity. Here, we neglect any effect of corruption on firms' capital and labor force. The marginal conditions are
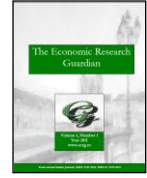
$$r(t) + \delta_k = \frac{\alpha_i \bar{\phi}_i F_i(t)}{K_i(t)}, \quad w_1(t) = \frac{\beta_i \bar{\phi}_i F_i(t)}{N_i(t)}. \tag{4}$$

The education sector

We specify the education sector's production function as follows

$$F_e(t) = A_e G^{\theta_e}(t) K_e^{\alpha_e}(t) N_e^{\beta_e}(t), \quad \alpha_e + \beta_e = 1, \quad A_e, \theta_e, \alpha_e, \beta_e > 0, \tag{5}$$

where A_e, θ_e, α_e and β_e are parameters. The marginal conditions are

$$r(t) + \delta_k = \frac{\alpha_e \, \bar{\phi}_e \, p(t) F_e(t)}{K_e(t)}, \quad w_1(t) = \frac{\beta_e \, \bar{\phi}_e \, p(t) F_e(t)}{N_e(t)}. \tag{6}$$

Workers' disposable income budget constraint

If there is no corruption on the worker, the current income is $r(t)\bar{k}(t) + w_1(t)h_1^{m_1}(t) + r(t)d_1(t)$. The total value of wealth that the representative household can sell to purchase goods and to save is $a_1(t)$. We assume that selling and buying wealth can be conducted instantaneously without any transaction cost. The disposable income is

$$\bar{k}_1(t) + d_1(t) + r(t)\bar{k}_1(t) + h_1^{m_1}(t)w_1(t)T_1(t) + r(t)d_1(t).$$

After paying the corruption fee to the officials, the representative household's disposable income $\hat{y}_1$ is

$$\hat{y}_1(t) \equiv \bar{\phi}_k R(t)a_1(t) + \bar{\phi}_w H_1(t)w_1(t)T_1(t), \tag{7}$$

where

$$a_1(t) = \bar{k}_1(t) + d_1(t), \quad H_1(t) \equiv h_1^{m_1}(t), \quad R(t) \equiv 1 + r(t).$$

The worker spends the disposable income on saving $s_1(t)$, consumption $c_1(t)$, and education $\tilde{T}_1(t)$. The budget constraint implies

$$c_1(t) + s_1(t) + p(t)\tilde{T}_1(t) = \hat{y}_1(t). \tag{8}$$

Equation means that consumption, education, and saving exhaust the consumer's disposable income. The (fixed) available time for work and leisure is denoted by T_0. We have the time constraint for each group as follows
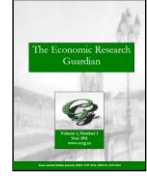
$$T_j(t) + \overline{T}_j(t) + \tilde{T}_j(t) = T_0, \quad j = 1, 2. \tag{9}$$

Insert (7) and (9) in (8)

$$c_1(t) + s_1(t) + \overline{w}_1(t)\overline{T}_1(t) + p_1(t)\tilde{T}_1(t) = \overline{y}_1(t) \equiv \bar{\phi}_k R(t)a_1(t) + \overline{w}_1(t)T_0, \tag{10}$$

where

$$\overline{w}_1(t) \equiv \bar{\phi}_w H_1(t)w_1(t), \quad p_1(t) \equiv p(t) + \overline{w}_1(t).$$

where $p_1(t)$ is the opportunity cost of the worker's education and $\overline{w}_1(t)$ is the worker's wage income.

Officials' disposable income and budget constraint
We assume that the representative official's wage income per unit of qualified work time is paid in proportion to the worker's wage rate per unit work time as follows

$$\overline{w}_2(t) = u_0 H_2(t) w_1(t), \tag{11}$$

where u_0 is determined by the government and $H_2(t) \equiv h_2^{m_2}(t)$. The official wage rate is proportional to the worker's market wage rate. The representative official receives the total income due to corruption as follows

$$w_c(t) \equiv \frac{\phi_i F_i(t) + \phi_e\, p(t) F_e(t) + \phi_k R(t) a_1(t) \overline{N}_1 + \phi_w H_1(t)(t) w_1(t) T_1(t) \overline{N}_1}{\overline{N}_2}. \tag{12}$$

The representative official's disposable income $\hat{y}_2$ is

$$\hat{y}_2(t) = R(t) a_2(t) + \overline{w}_2(t) T_2(t) + w_c(t). \tag{13}$$

The official spends the disposable income on saving $s_2(t)$, education $\tilde{T}_2(t)$, and consumption $c_2(t)$. We have the budget constraint as follows

$$c_2(t) + s_2(t) + p(t)\tilde{T}_2(t) = \hat{y}_2(t). \tag{14}$$
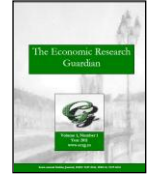
Insert (9) and (13) in (14)

$$c_2(t) + s_2(t) + \overline{w}_2(t)\overline{T}_2(t) + p_2(t)\tilde{T}_2(t) = \overline{y}_2(t) \equiv R(t) a_2(t) + \overline{w}_2(t) T_0 + w_c(t). \tag{15}$$

where $p_2(t) \equiv \overline{w}_2(t) + p(t),$ which is the opportunity cost of the official's education.

The utility functions and optimal decisions
The representative household in each group chooses four variables, $c_j(t)$, $s_j(t)$, $\overline{T}_j(t)$, and $\tilde{T}_j(t)$ subject to the budget constraints. We apply Zhang's utility function $U_j(t)$ as follows

$$U_j(t) = u_j(G(t),t)\overline{T}_j^{\sigma_{0j}}(t)\tilde{T}_j^{\eta_{0j}}(t)c_j^{\xi_{0j}}(t)s_j^{\lambda_{0j}}(t), \quad \sigma_{0j}, \eta_{0j}, \xi_{0j}, \lambda_{0j} > 0,$$

where u_j is a time-dependent variable, σ_{0j}, η_{0j}, ξ_{0j}, and λ_{0j} are called respectively the worker's/official's propensity to stay at home, to receive education, to consume good, and to hold wealth. Maximizing U_j subject to budget constraint (10)/(14) yields

$$\overline{w}_j(t)\overline{T}_j(t) = \sigma_j\, \overline{y}_j(t), \quad p_j(t)\widetilde{T}_j(t) = \eta_j\, \overline{y}_j(t), \quad c_j(t) = \xi_j\, \overline{y}_j(t), \quad s_j(t) = \lambda_j\, \overline{y}_j(t), \qquad (16)$$

where

$$\rho_j \equiv \frac{1}{\sigma_{0j} + \eta_{0j} + \xi_{0j} + \lambda_{0j}}, \quad \sigma_j \equiv \rho_j\,\sigma_{0j}, \quad \eta_j \equiv \rho_j\,\eta_{0j}, \quad \xi_j \equiv \rho_j\,\xi_{0j}, \quad \lambda_j \equiv \rho_j\,\lambda_{0j}.$$

The wealth accumulation

According to the definitions of $s_j(t)$ and $a_j(t)$, the change in the household's wealth is given by

$$\dot{a}_j(t) = s_j(t) - a_j(t). \tag{17}$$

This equation tells that change in the wealth is equal to saving minus dissaving.

The public sector

All the government's tax income is spent on the public sector and paying government debts. We assume that public services are dependent only on the officials' labor input $N_2(t)$. We describe the public sector production function as follows

$$G(t) = A_p\, N_2^{\gamma}(t), \quad A_p, \gamma > 0. \tag{18}$$
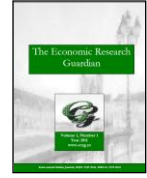
The government's tax income is

$$I_p(t) = \tau_i\, F_i(t) + \tau_e\, p(t)F_e(t). \tag{19}$$

The government's expenditure is

$$Y_p(t) = \overline{w}_2(t)T_2(t)\overline{N}_2. \tag{20}$$

The government budget

Following Diamond (1965) and Barro (1974), we assume that the government issues an amount debt $D(t)$, which may be considered as real-valued bonds. It is supposed that asset holders regard equity and government bonds as perfect substitutes. The debt pays the amount of real interest $r(t)D(t)$. The government finances current spending by collecting taxes and issuing interest-bearing debt. The dynamics of public debt is

$$\dot{D}(t) = r(t)D(t) + Y_p(t) - I_p(t). \tag{21}$$

As the debt is owned by households, we have

$$D(t) = d_1(t)\overline{N}_1 + d_2(t)\overline{N}_2. \tag{22}$$

For simplicity of analysis we don't introduce possible corruption of the officials on governance and government expenditures. The interaction between corruption and governance may affect the efficiency of public spending, which will influence growth (Dzhumashev, 2014).

Wealth being owned by the households and full employment of capital
The wealth owned by the two groups and full employment of capital imply

$$\overline{k}_1(t)\overline{N}_1 + \overline{k}_2(t)\overline{N}_2 = K(t), \quad K_i(t) + K_e(t) = K(t). \tag{23}$$

The demand equaling the supply
Change in physical capital is the output of physical good minus the total consumption and depreciation, i.e.

$$\dot{K}(t) = F_i(t) + \sum_{j=1}^{2} c_j(t)\overline{N}_j - \delta_k K(t). \tag{24}$$

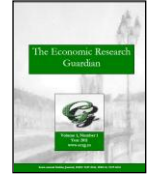Demand for education and supply of education balances

$$\sum_{j=1}^{2} \tilde{T}_j(t)\overline{N}_j = F_e(t). \tag{25}$$

Accumulation of human capital
In our approach, human capital accumulation is through education. It costs household money and time as just modelled. We follow Uzawa (1965) in modelling human capital accumulation. We generalize Uzawa's human capital accumulation as follows

$$\dot{h}_j(t) = \frac{\upsilon_{ej}\left(F_e(t)/\overline{N}\right)^{a_{ej}}\left(h_j^{m_j}(t)\tilde{T}_{ej}(t)\right)^{b_{ej}}}{h_j^{\pi_{ej}}(t)} - \delta_{hj} h_j(t), \tag{26}$$

where δ_{hj} (> 0) is the depreciation rate of human capital, υ_{ej}, m_{ej}, a_{ej}, and b_{ej} are non-negative parameters. The sign of π_e may be negative or positive. The equation implies that human capital rises in education service per unit time, $F_e(t)/T_e(t)\overline{N}$, and in the (qualified) total study time, $\left(H^m(t)T_e(t)\right)^{b_e}$.

The term $1/H^{\pi_e}$ implies that learning through education may exhibit increasing returns to scale in the case of $\pi_e < 0$ or decreasing returns to scale in the case of $\pi_e > 0$.

We built the model. The dynamic general equilibrium model describes the interdependence between wealth accumulation and human capital accumulation of the workers and officials, public good supply, income and wealth distribution, endogenous labor supply, and economic structure with different ways of corruption. The rest of the paper examines properties of the model.

3. The behavior of the model

This section examines properties of the nonlinear dynamic model. Although there are many nonlinear interactions in the model, we show that we can simply follow its motion with computer by providing a computational procedure. To describe the procedure, we first introduce a variable

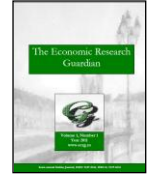$$z(t) \equiv \frac{r(t) + \delta_k}{w_1(t)}.$$

We prove the following lemma, demonstrating how to follow the motion of the economic system. The following lemma is checked in the Appendix.

Lemma
The dynamics of the economic system is governed by the following five differential equations with five variables, $z(t)$, $G(t)$, $a_2(t)$, $h_1(t)$, and $h_2(t)$

$$\dot{z}(t) = \Omega_z(z(t), G(t), h_1(t), h_2(t), a_2(t)),$$
$$\dot{G}(t) = \Omega_G(z(t), G(t), h_1(t), h_2(t), a_2(t)),$$
$$\dot{a}_2(t) = \Omega_2(z(t), G(t), h_1(t), h_2(t), a_2(t)),$$
$$\dot{h}_j(t) = \Omega_{hj}(z(t), G(t), h_1(t), h_2(t), a_2(t)), \quad j = 1, 2, \tag{27}$$

where $\Omega_x(t)$ is are functions of $z(t)$, $G(t)$, $a_2(t)$, $h_1(t)$, and $h_2(t)$ defined in the appendix. The values of all the other variables are uniquely determined as functions of $z(t)$, $G(t)$, $a_2(t)$, $h_1(t)$, and $h_2(t)$ at any point in time by the following procedure: $r(t)$ and $w_1(t)$ by (A2) $\rightarrow$ $p(t)$ by (A4) $\rightarrow$ $\overline{w}_1(t)$ by definition $\rightarrow$ $\overline{w}_2(t)$ by (11) $\rightarrow$ $p_j(t)$ by definition $\rightarrow$ $a_1(t)$ by (A22) $\rightarrow D(t)$ by (A21) $\rightarrow$ $\overline{y}_2(t)$ by (A20) $\rightarrow \overline{y}_1(t)$ by (A5) $\rightarrow \overline{y}_j(t)$ by (A5) $\rightarrow c_j(t)$, $s_j(t)$, $\tilde{T}_j(t)$ and $\overline{T}_j(t)$ by (16) $\rightarrow T_1(t)$ by (A7) $\rightarrow T_2(t)$ by (A18) $\rightarrow N_j(t)$ by (1) $\rightarrow K_j(t)$ by (A13) $\rightarrow N_j(t)$ by (A1) $\rightarrow F_j(t)$ by (A3) $\rightarrow I_p(t)$ by (19) $\rightarrow Y_p(t)$ by (20) $\rightarrow w_c(t)$ by (12).

We don't explicitly present the expressions as they are tedious and not easy to interpret the analytical expressions. For illustration, we simulate the model to demonstrate dynamic properties of the model. The parameter values are taken on the following values

$$\overline{N}_1 = 100, \ \overline{N}_2 = 10, \ T_0 = 24, \ u_0 = 1.5, \ \alpha_i = 0.3, \ \alpha_e = 0.32, \ A_i = 1.2, \ A_e = 1,$$
$$A_p = 1.5, \ \gamma = 0.6, \ m_1 = 0.5, \ m_2 = 0.6, \ \theta_i = 0.3, \ \theta_e = 0.6, \ \tau_i = 0.06, \ \tau_e = 0.04,$$
$$\xi_{01} = 0.15, \ \lambda_{01} = 0.7, \ \eta_{01} = 0.05, \ \sigma_{01} = 0.2, \ \xi_{02} = 0.15, \ \lambda_{02} = 0.8, \ \eta_{01} = 0.06,$$
$$\sigma_{02} = 0.18, \ \theta = 0.1, \ \gamma = 0.3, \ \tau = 0.01, \ \phi_i = 0.1, \ \phi_e = 0.1, \ \phi_k = 0.086,$$
$$\phi_w = 0.085, \ b_{e1} = 0.35, \ b_{e2} = 0.35, \ v_{e1} = 0.5, \ v_{e2} = 0.6, \ \pi_{e1} = 0.4, \ \pi_{e1} = 0.3,$$
$$\delta_k = 0.05, \ \delta_{h1} = 0.05, \ \delta_{h1} = 0.045. \tag{28}$$

The ratio between the workers and the officials is $10:1$. The representative worker's relative propensity to save and the official's relative propensity to save are respectively
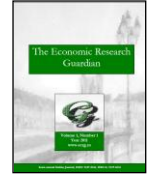
$$\lambda_1 = \frac{0.7}{0.7 + 0.2 + 0.05 + 0.15} = 0.7 > \lambda_2 = \frac{0.8}{0.8 + 0.15 + 0.18 + 0.06} = \frac{8}{11.9}.$$

The official's propensity to save is lower than the worker's propensity to save. Intuitively this implies that in this neoclassical growth model corruption may harm economic growth as the wealth is shifted from the group with the higher propensity to save to the group with lower propensity to save. The representative worker's relative propensity to receive education and the official's relative propensity to receive education are respectively

$$\eta_1 = \frac{0.05}{0.7 + 0.2 + 0.05 + 0.15} = 0.05 < \eta_2 = \frac{0.06}{0.8 + 0.15 + 0.18 + 0.06} = \frac{6}{119}.$$

The worker's propensity to receive education is slightly lower than the official's. The depreciation rates of human capital and physical capital are approximately 0.05. Human capital accumulations of the two groups exhibit decreasing return to scale. The government fixes the wage rate at 1.5. The official gets higher wage income per unit of time if the two groups have the same level of human capital. The tax rates on the production sector and education sector are respectively 6 and 4 percent. We fix the different corruption rates approximately 10 percent. Under (28) we identify two equilibrium points. The variable values at the equilibrium point with negative government debt are

$$r = -0.007, \ w_c = 1239, \ w_1 = 7.03, \ \overline{w}_1 = 19.9, \ \overline{w}_2 = 67.7, \ p = 1.05,$$
$$p_1 = 20.96, \ p_2 = 68.8, \ h_1 = 9.6, \ h_2 = 22.2, \ N_1 = 3472, \ N_2 = 148.4,$$
$$K_i = 243562, \ K_e = 2034, \ F_i = 41195, \ F_e = 299, \ Y = 41509.7, \ I_p = 2484,$$

$$Y_p = 1565, \ \ G = 30.1, \ \ D = -124547, \ \ a_1 = 719, \ \ a_2 = 4914, \ \ c_1 = 154.1,$$

$$c_2 = 921.3, \ \ T_1 = 11.2, \ \ \overline{T}_1 = 10.3, \ \ \tilde{T}_1 = 2.45, \ \ T_2 = 2.31, \ \ \overline{T}_2 = 16.3, \ \ \tilde{T}_2 = 5.36. \tag{29}$$

The rate of interest and debt are negative at this equilibrium. It should be remarked that the net cost of capital for producers positive, that is

$$r + \delta_k = -0.007 + 0.05 > 0.$$

The five eigenvalues at this equilibrium is

$$\{-1.815, \ -0.416, \ -0.061, \ -0.05, \ -0.024\}.$$

The stability is important as it implies that the system will converge to the equilibrium point listed in (29). We specify the initial conditions near the equilibrium point as follows

$$z(0) = 0.0058, \ \ G(0) = 31, \ \ h_1(0) = 9.4, \ \ h_2(0) = 22, \ \ a_2(0) = 4900. \tag{30}$$

We plot the motion of the system near this point in Figure 1. We see that the system moves toward the equilibrium point, some variables rising and some variables falling from the initial point.
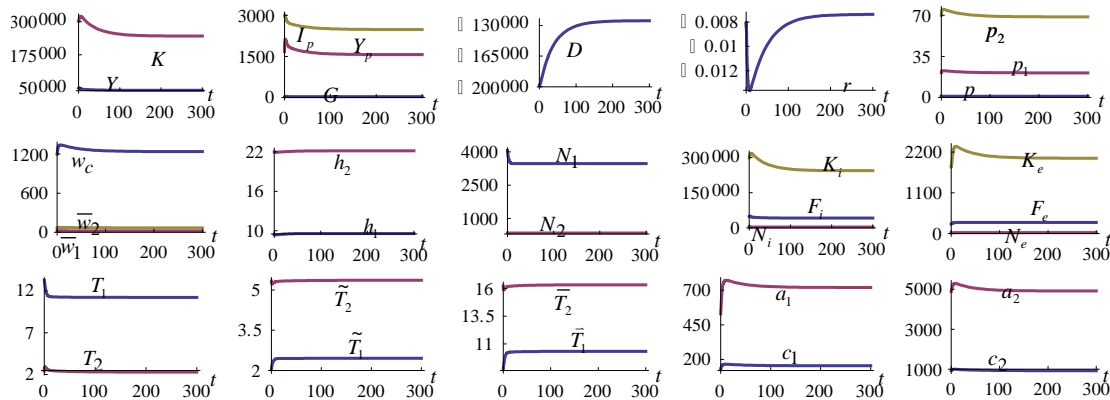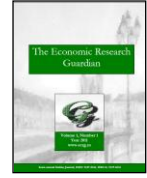

Figure 1 - The motion of the system with negative government debts

The variable values at the equilibrium point with positive government debt are

$$r = 0.056, \ \ w_c = 161.3, \ \ w_1 = 0.9, \ \ \overline{w}_1 = 2.44, \ \ \overline{w}_2 = 8.6, \ \ p = 1.12,$$

$$p_1 = 3.56, \ \ p_2 = 9.67, \ \ h_1 = 8.8, \ \ h_2 = 21.6, \ \ N_1 = 3178, \ \ N_2 = 0.23,$$

$$K_i = 10898, \ \ K_e = 721, \ \ F_i = 4594, \ \ F_e = 248.6, \ \ Y = 4871.7, \ \ I_p = 286.7,$$

$$Y_p = 0.31, \ \ G = 0.62, \ \ D = 5094.9, \ \ a_1 = 96.7, \ \ a_2 = 704.1, \ \ c_1 = 20.7, \ \ c_2 = 132,$$

$$T_1 = 10.75, \ \ \overline{T}_1 = 11.3, \ \ \tilde{T}_1 = 1.94, \ \ T_2 = 0.004, \ \ \overline{T}_2 = 18.5, \ \ \tilde{T}_2 = 5.5. \tag{31}$$

The national output at this equilibrium is lower than the equilibrium point with negative debt. The eigenvalues at this equilibrium point are

$$\{-0.362, \ -0.069, \ -0.043, \ 0.039, \ 0.026\}.$$

The system at this equilibrium is unstable. This implies that comparative dynamic analysis is invalid over long period of time. We specify the initial conditions near the unstable equilibrium point as follows

$$z(0) = 0.12, \ G(0) = 0.62, \ h_1(0) = 8.8, \ h_2(0) = 21, \ a_2(0) = 704. \qquad (32)$$

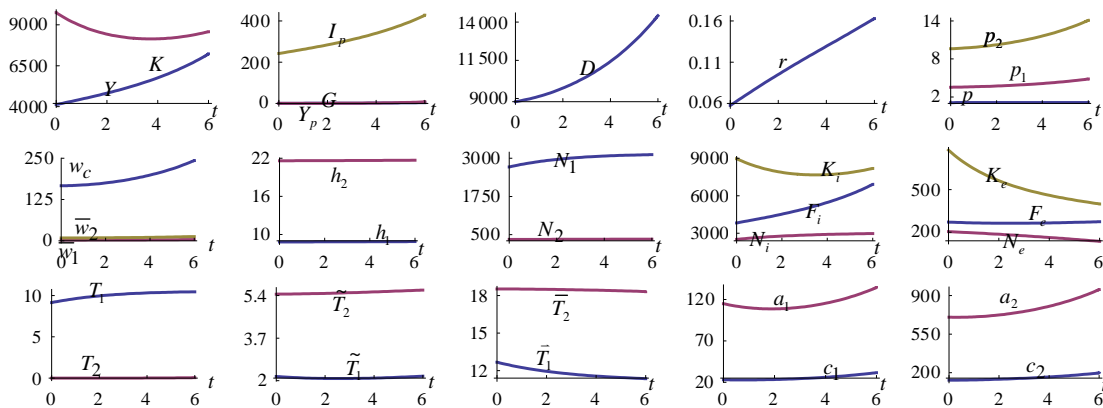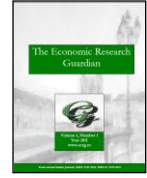The changes of the variables over time are plotted in Figure 2.

Figure 2 - The motion of the system with positive government debts

As the equilibrium point is unstable, it informs us that comparative dynamic analysis cannot be effectively conducted. We follow the motion of the economy for a short period of time in Figure 2. The debt rises without limit. It is well known that some neoclassical growth models with debts have saddle points (e.g., Turnovsky and Sen, 1991, Lin, 2000). The equilibrium point in this study has similar features as in these models. It is significant to plot shifts of paths with changes in public policies and other parameters because the focus on unstable steady states does not provide enough information for describing behavior of the system. Michel et al. (2010: 925) describes: "To operationalize the notion of unstable government debt dynamics, we consider steady states which are locally unstable under the assumption of a permanently balanced primary budget. However, the economy can be stabilized at these steady states if one allows for appropriate budgetary adjustments. For tractability, we consider debt stabilizing rules that specify these adjustments as a linear function of the two state variables of the model (physical capital and real government debt). Moreover, we assume that such adjustments can be brought about by two different instruments (government consumption or a lump-sum tax on young agents)." This study will not examine how to stabilize the equilibrium point with low national income and high government debt. We are concerned with the impact of changes in some parameters on the two possible paths of the economic development.

4. Comparative dynamic analysis

The previous section plotted the motion of the economic system with the initial condition. This section shows how a change in any parameter alters the motion of the system. As the system has two equilibrium points and one is stable and the other one is unstable, our comparative dynamic analysis are effective only locally. We use the variable, $\overline{\Delta}x(t)$, to represent the change rate of the variable, $x(t)$, in percentage due to a change in the parameter value.

4.1. A fall in the corruption rate on the industrial sector

We first study how the economic system is affected when the corruption rate on the industrial sector is reduced as follows: $\phi_i : 0.1 \Rightarrow 0.098$, where "$\Rightarrow$" stands for "being changed to". The effects on the stable equilibrium point are plotted in Figure 3. The national capital stock rises initially and changes slightly in the long term. The national output rises. The official's and worker's wage rates per unit time, $\overline{w}_2$ and $\overline{w}_1$, rise over time. The corrupt income per official w_c is increased. The government debt rises initially and changes slightly in the long term. The government tax income, expenditure and service are all increased. The rate of interest rises initially and falls in the long term. The education fee and the opportunity costs of the two groups rise. The official's human capital falls and the worker's human capital rises. The officials' total labor supply changes slightly and the workers' total labor supply rises. The industrial sector employs more labor force and the education sector employs less labor force. The two sectors employ more capital stocks. The industrial sector expands. The education sector shrinks initially and changes slightly in the long term. The officials work longer hours and the workers slightly change their work hours. The officials reduce their leisure and education hours. In the long term both the officials and workers increase their consumption and wealth. Examining the effects of various variables on the officials' wealth and income, we see that it is mainly due to the increased work hours that enables the official to own more wealth and consumes more.
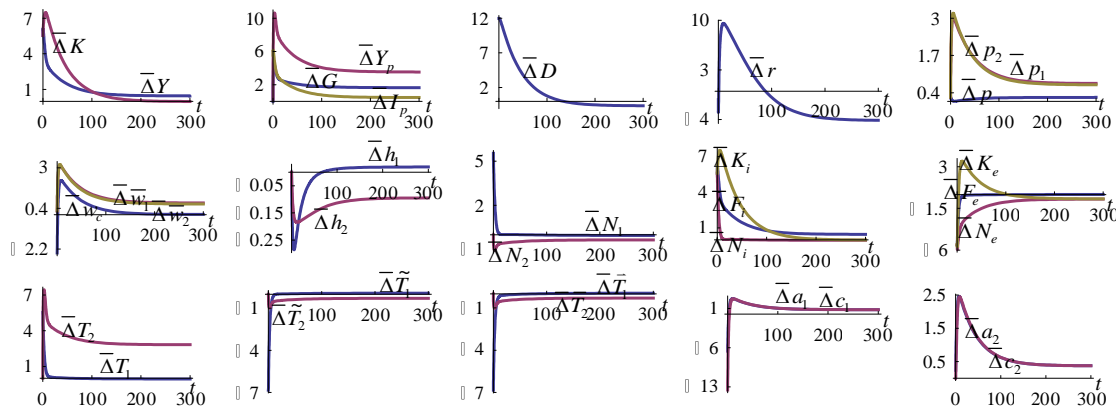

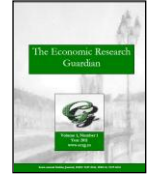Figure 3 - Reduce corruption rate on industrial sector near path

Figure 3 plots the impact on the stable equilibrium point. We describe the effects on the unstable equilibrium point in Figure 4. As the equilibrium point is unstable, we can follow the system only in a very short period. Comparing Figures 3 and 4, we see that the directions of the effects are different in the two cases.
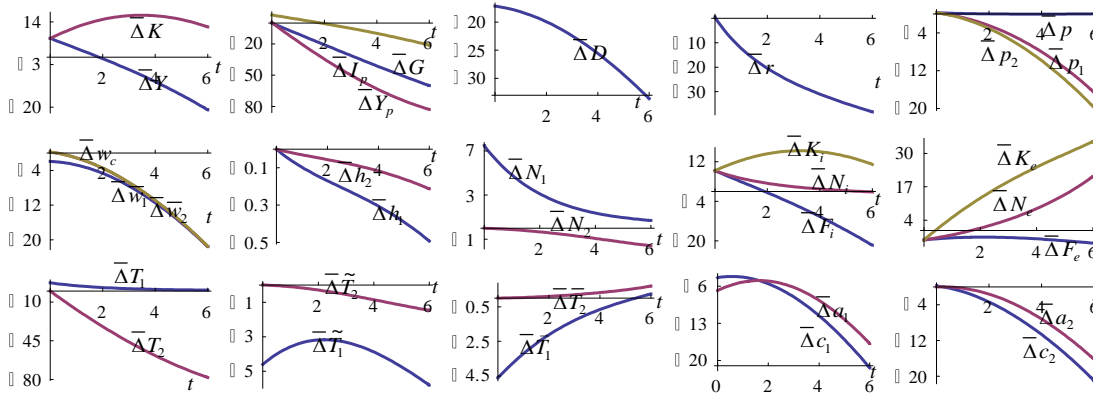

Figure 4 - Reduce corruption rate on industrial sector near unstable path

The effects on the unstable equilibrium point are listed in (30).

$$\overline{\Delta} r = -1.3, \ \overline{\Delta} w_c = 28.6, \ \overline{\Delta} w_1 = 28.5, \ \overline{\Delta} \overline{w}_1 = 30.4, \ \overline{\Delta} \overline{w}_2 = 29.9, \ \overline{\Delta} p = -0.28,$$
$$\overline{\Delta} p_1 = 20.7, \ \overline{\Delta} p_2 = 26.4, \ \overline{\Delta} h_1 = 3, \ \overline{\Delta} h_2 = 1.85, \ \overline{\Delta} N_1 = 0.16, \ \overline{\Delta} N_2 = 158.3,$$
$$\overline{\Delta} K_i = 31, \ \overline{\Delta} K_e = 7.03, \ \overline{\Delta} F_i = 29.7, \ \overline{\Delta} F_e = 6.6, \ \overline{\Delta} Y = 28.4, \ \overline{\Delta} I_p = 28.8,$$
$$\overline{\Delta} Y_p = 232, \ \overline{\Delta} G = 76.7, \ \overline{\Delta} D = 30.3, \ \overline{\Delta} a_1 = 30.2, \ \overline{\Delta} a_2 = 29, \ \overline{\Delta} c_1 = 30.2,$$
$$\overline{\Delta} c_2 = 29, \ \overline{\Delta} T_1 = -1.3, \ \overline{\Delta} \overline{T}_1 = -0.11, \ \overline{\Delta} \tilde{T}_1 = 7.9, \ \overline{\Delta} T_2 = 155.5, \ \overline{\Delta} \overline{T}_2 = -0.65,$$
$$\overline{\Delta} \tilde{T}_2 = 2.1. \tag{33}$$

4.2. A rise in the corruption rate on the worker's wage income

We examine how the economic system is affected when the corruption rate on the wage rate is enhanced as follows: $\phi_w: 0.085 \Rightarrow 0.086$. The effects on the stable equilibrium point are plotted in Figure 5. The national capital stock falls. The national income falls initially and rises in the long term. The official's and worker's wage rates per unit time and official's corrupt income fall over time. The government debt falls initially and rises slightly in the long term. The government tax income, expenditure and service are all reduced. The rate of interest rises initially, then falls, and rise in the long term. The education fee changes slightly and the opportunity costs of the two groups fall. The official's human capital rises. The worker's human capital rises initially and falls in the long term. The officials' total labor supply rises. The workers' total labor supply falls initially and changes slightly. The worker's time distribution changes slightly in the long term. The official works less hours, has longer leisure time, and spends more time on education. The officials and workers own less wealth and consume less in the long term. The education sector's output changes slightly. The industrial sector's output is reduced.
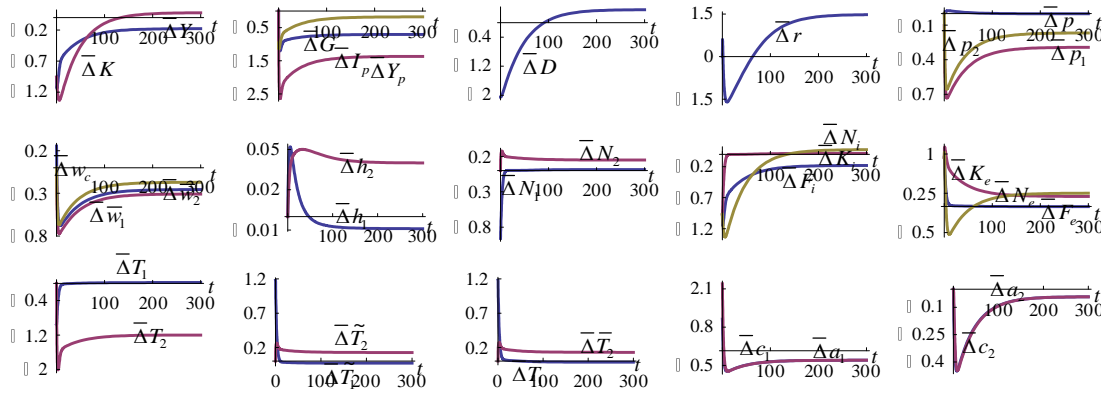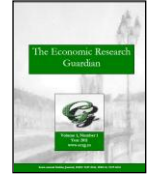
Figure 5 - Raise corruption rate on worker's wage income on stable path

We describe the effects on the unstable equilibrium point in Figure 6.
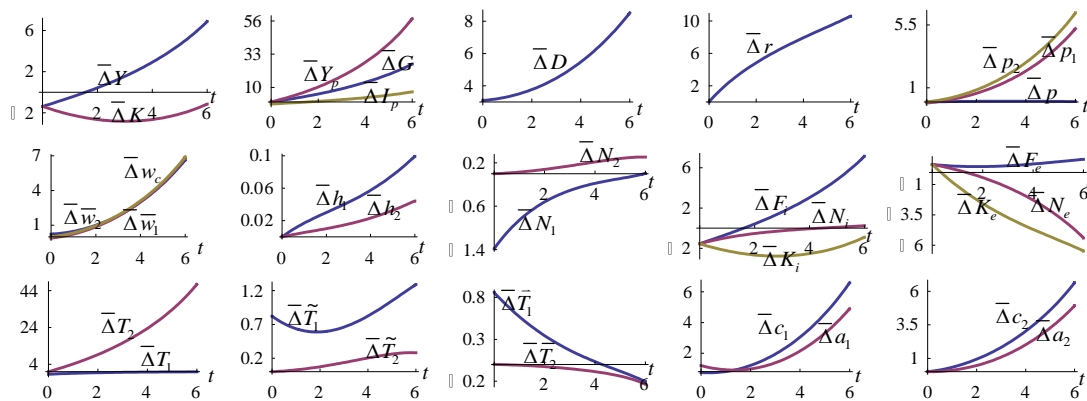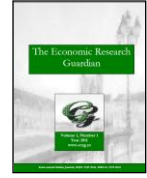

Figure 6 - Raise corruption rate on worker's wage income near unstable path

The effects on the unstable equilibrium point are listed in (31).

$$\overline{\Delta} r = 0.6, \ \overline{\Delta} w_c = -9.6, \ \overline{\Delta} w_1 = -9.2, \ \overline{\Delta}\overline{w}_1 = -9.9, \ \overline{\Delta}\overline{w}_2 = -9.6, \ \overline{\Delta} p = 0.2,$$

$$\overline{\Delta} p_1 = -6.7, \ \overline{\Delta} p_2 = -8.5, \ \overline{\Delta} h_1 = -1.3, \ \overline{\Delta} h_2 = -0.8, \ \overline{\Delta} N_1 = -0.11, \ \overline{\Delta} N_2 = -30.9,$$

$$\overline{\Delta} K_i = -10, \ \overline{\Delta} K_e = -2.9, \ \overline{\Delta} F_i = -9.7, \ \overline{\Delta} F_e = -2.8, \ \overline{\Delta} Y = -9.3, \ \overline{\Delta} I_p = -9.4,$$

$$\overline{\Delta} Y_p = -37.3, \ \overline{\Delta} G = -19.9, \ \overline{\Delta} D = -9.9, \ \overline{\Delta} a_1 = -9.8, \ \overline{\Delta} a_2 = -9.4, \ \overline{\Delta} c_1 = -9.8,$$

$$\overline{\Delta} c_2 = -9.4, \ \overline{\Delta} T_1 = 0.6, \ \overline{\Delta}\overline{T}_1 = 0.05, \ \overline{\Delta}\tilde{T}_1 = -3.35, \ \overline{\Delta} T_2 = -30.6, \ \overline{\Delta}\overline{T}_2 = 0.29,$$

$$\overline{\Delta}\tilde{T}_2 = -0.96. \tag{34}$$

4.3. A rise in the worker population

We now study the impact of a rise in the worker population on the economic system as follows: $\bar{N}_1 : 100 \Rightarrow 101$. The effects on the stable equilibrium point are plotted in Figure 7. The national capital stock and national output fall initially and rise in the long term. The official's and worker's wage rates per unit time fall. The official's corrupt income rises over time. The government debt falls initially and rises in the long term. The government tax income falls initially and is enhanced in the long term. The government expenditure and service are decreased. The rate of interest rises initially, then falls, and rises in the long term. The education fee changes slightly. The opportunity costs of the two groups are lowered. The official's human capital rises. The worker's human capital rises initially and falls in the long term. The officials' and workers' total labor supplies are increased. The worker's time distribution changes little in the long term. The official works less hours, has longer leisure time, and spends more time on education. The officials and workers own less wealth and consume less in the long term. The education sector's output is augmented. The industrial sector's output falls initially and is increased in the long term.
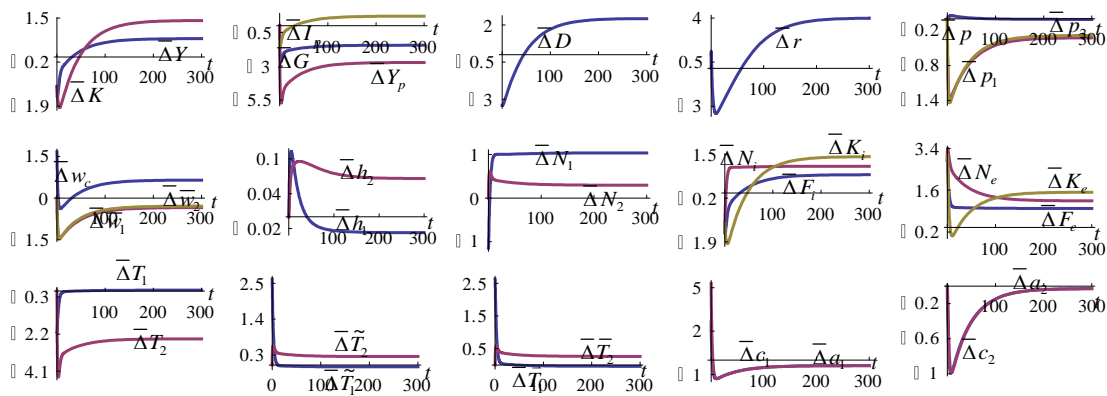

Figure 7 - Rise in worker population near stable path

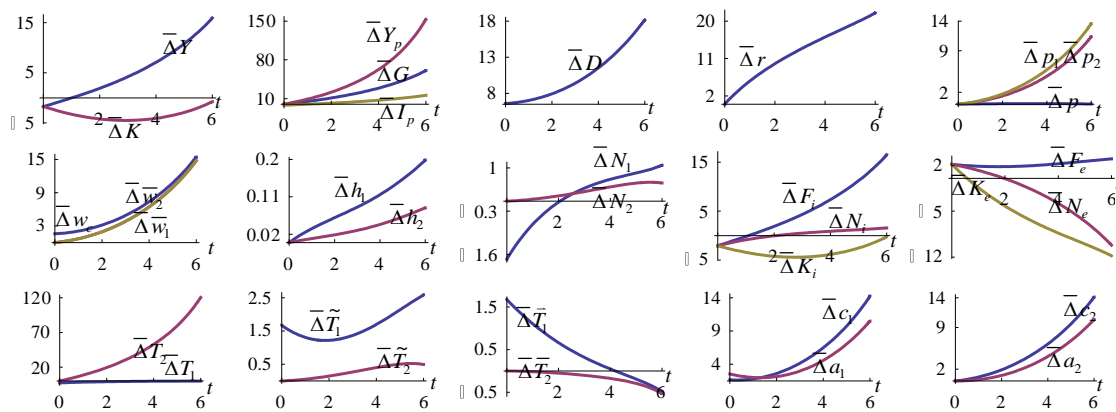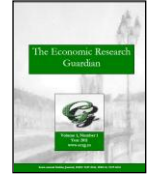We describe the effects on the unstable equilibrium point in Figure 8.

Figure 8 - Rise in worker population near unstable path

The effects on the unstable equilibrium point are listed in (32).

$$\overline{\Delta}\,r = 1.6,\ \ \overline{\Delta}\,w_c = -18.9,\ \ \overline{\Delta}\,w_1 = -19,\ \ \overline{\Delta}\,\overline{w}_1 = -20,\ \ \overline{\Delta}\,\overline{w}_2 = -20,\ \ \overline{\Delta}\,p = 0.44,$$
$$\overline{\Delta}\,p_1 = -13.7,\ \ \overline{\Delta}\,p_2 = -17.6,\ \ \overline{\Delta}\,N_1 = -3,\ \ \overline{\Delta}\,N_2 = -1.9,\ \ \overline{\Delta}\,K_i = 0.68,\ \ \overline{\Delta}\,K_e = -55,$$
$$\overline{\Delta}\,F_i = -20,\ \ \overline{\Delta}\,F_e = -5.9,\ \ \overline{\Delta}\,Y = -19.3,\ \ \overline{\Delta}\,I_p = -5.5,\ \ \overline{\Delta}\,Y_p = -18.5,\ \ \overline{\Delta}\,G = -18.7,$$
$$\Delta D = -63.8,\ \ \overline{\Delta}\,a_1 = -38.3,\ \ \overline{\Delta}\,a_2 = -19.9,\ \ \overline{\Delta}c_1 = -20.1,\ \ \overline{\Delta}c_2 = -19.4,$$
$$\overline{\Delta}T_1 = 1.2,\ \ \overline{\Delta}\overline{T}_1 = 0.13,\ \ \overline{\Delta}\tilde{T}_1 = -7.4,\ \ \overline{\Delta}T_2 = -54.8,\ \ \overline{\Delta}\overline{T}_2 = 0.66,\ \ \overline{\Delta}\tilde{T}_2 = -2.2. \qquad (35)$$

4.4. Raise the official's wage ratio

We examine the effects of the following rise in the official's wage ratio on the economic system: $u_0 : 1.5 \Rightarrow 1.51$. The effects on the stable equilibrium point are plotted in Figure 9. The national capital stock and national output are enhanced initially and are affected slightly in the long term. The government debt rises initially and changes slightly in the long term. The rate of interest rises initially and falls in the long term. The government tax income, expenditure and service are all augmented. The official's and worker's wage rates per unit time and the official's corrupt income are enhanced. The education fee is slightly affected. The opportunity costs of the two groups are augmented. The official's human capital falls initially and changes slightly in the long term. The worker's human capital is reduced. The officials' and workers' total labor supplies are little changed in the long term. The two sectors are changed slightly in the long term. The official owns more wealth and consumes more. The worker's long-run wealth and consumption levels are slightly affected.
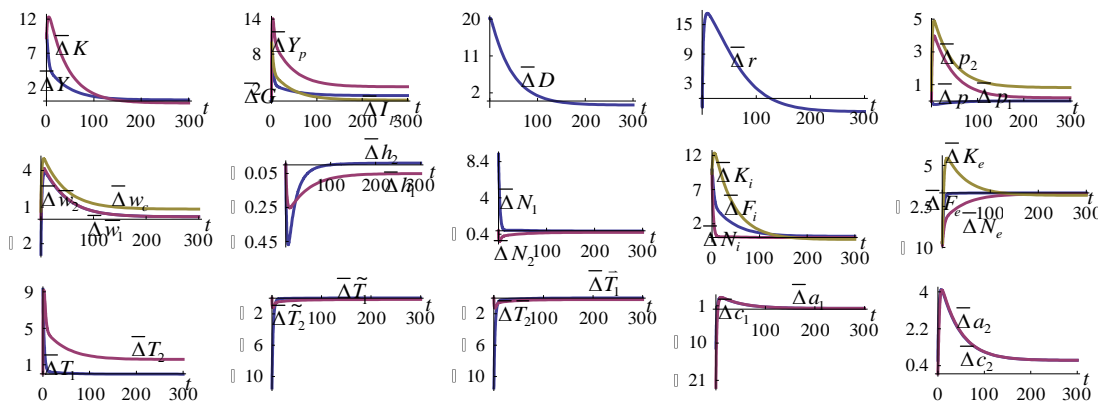

Figure 9 - Raise official's wage ratio near stable path

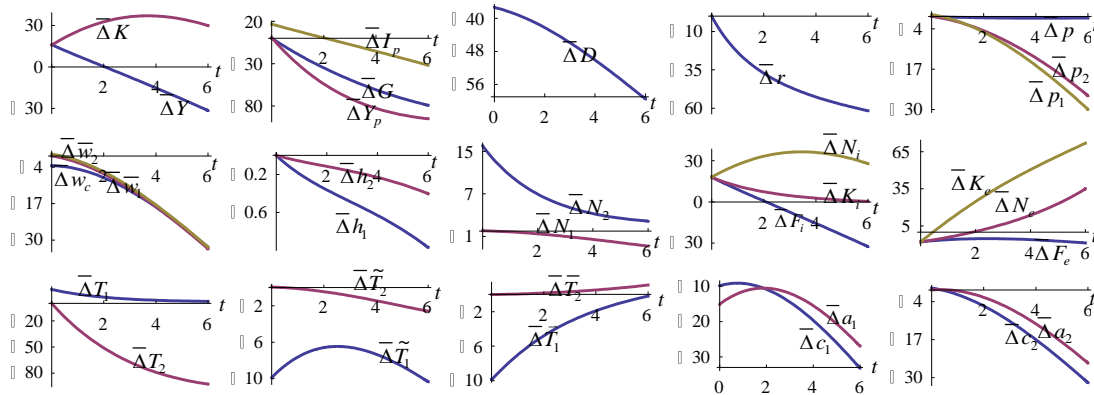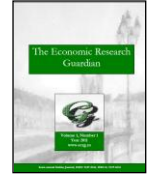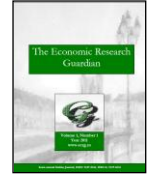We describe the effects on the unstable equilibrium point in Figure 10.

Figure 10 - Raise official's wage ratio near unstable path

The effects on the unstable equilibrium point are listed in (33).

$$\overline{\Delta}\, r = -1.15, \quad \overline{\Delta}\, w_c = 18.6, \quad \overline{\Delta}\, w_1 = 17.9, \quad \overline{\Delta}\, \overline{w}_1 = 19.1, \quad \overline{\Delta}\, \overline{w}_2 = 19.6, \quad \overline{\Delta}\, p = -0.34,$$

$$\overline{\Delta}\, p_1 = 13, \quad \overline{\Delta}\, p_2 = 17.3, \quad \overline{\Delta}\, h_1 = 2, \quad \overline{\Delta}\, h_2 = 1.3, \quad \overline{\Delta}\, N_1 = 0.15, \quad \overline{\Delta}\, N_2 = 87.9,$$

$$\overline{\Delta}\, K_i = 19.7, \quad \overline{\Delta}\, K_e = 4.8, \quad \overline{\Delta}\, F_i = 19, \quad \overline{\Delta}\, F_e = 4.5, \quad \overline{\Delta}\, Y = 18.1, \quad \overline{\Delta}\, I_p = 18.4,$$

$$\overline{\Delta}\, Y_p = 123.1, \quad \overline{\Delta}\, G = 46, \quad \overline{\Delta}\, D = 19.6, \quad \overline{\Delta}\, a_1 = 19, \quad \overline{\Delta}\, a_2 = 19.1, \quad \overline{\Delta}\, c_1 = 19, \quad \overline{\Delta}\, c_2 = 19.1,$$

$$\overline{\Delta}\, T_1 = -0.86, \quad \overline{\Delta}\, \overline{T}_1 = -0.1, \quad \overline{\Delta}\, \tilde{T}_1 = 5.3, \quad \overline{\Delta}\, T_2 = 86.5, \quad \overline{\Delta}\, \overline{T}_2 = -0.5, \quad \overline{\Delta}\, \tilde{T}_2 = 1.5. \qquad (36)$$

5. Conclusion

This study proposed a dynamic general equilibrium model with endogenous wealth, human capital, and corruption, and government debt. It is a synthesis of Solow's gowth model, Uzawa's two-sector model, Diamond's government debt model, Uzawa-Lucas' human capital growth model with Zhang's approach to household behavior. The population is classified into workers and officials. The economy is composed of one (private) production sector, one education sector, and one public sector. Production and education sectors are perfectly competitive. Public service is supplied by the public sector finally supported by the government's tax income with a sole input factor, officials. Public services affect productivities of the production and education sectors and people's welfare. Officials are corrupt in the sense that they take bribes from the private sectors and households. Corruption is measured by corruption rates on the output level of the production sector, wealth interest returns and wage income of workers. We simulated the model and carried out comparative dynamic analysis to see transitory and long-run effects of changes in some parameters. The economic dynamic with the specified parameters values have two equilibrium points, the one with negative government debt being stable and the other one with positive debt being unstable. We carried out comparative dynamic analysis separately with regard to the two equilibrium points. The comparative dynamic analysis provided insights into the complicated impact of corruption on economic growth process. It also has some implications on

political decision makings. For instance, in order to reduce corruption a government may increase officials' wage rate.

Our result implies that the efficiency of this commonly practiced method in reducing corruption is situation-dependent. If the system is stable, then the policy encourages corruption as the corrupt officials get more income and gaps of consumption and wealth between the workers and the officials are enlarged. If the system is currently near the unstable equilibrium point, the policy harms both the officials and workers even though the government appears cleaner in the sense that the corrupt income is reduced. It should be noted that our model may be generalized and extended in different ways. Our comparative dynamic analysis is limited to a few cases. We might get more insights from further simulation. There is a vast literature on corruption. But there are relatively few macroeconomic growth models with endogenous wealth and human capital in a dynamic general equilibrium framework. Not all officials are corrupt. It is important to examine co-existence of corrupt and clean officials and their mutual-dependent impact on societies. Another important issue is possible existence of multiple channels and forms of corruption. Dynamic interdependence between institutional structures, economic development and corruption networks. For instance, corruption may discourage innovative activities as innovators need government's support (e.g., Mo, 2001).
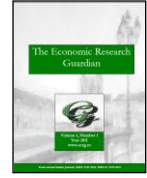
Acknowledgements

References

Alonso-Carrera J, Freire-Sere MJ (2004). Multiple Equilibria, Fiscal Policy and Human Capital Accumulation. *Journal of Economic Dynamics and Control*. 28(4): 841-56.

Barro RJ (1974). Are Government Bonds Net Wealth? *Journal of Political Economy*. 82(6): 1095-1017.

Diamond P (1965). National Debt in a Neoclassical Growth Model. American Economic Review 55, 1126–1150.

Barro R, Mankiw NG, Sala-i-Martin X (1995). Capital Mobility in Neoclassical Models of Growth. *American Economic Review*. 85(1): 103-116.

Barro RJ, Sala-i-Martin X (1995). *Economic Growth*. McGraw-Hill, Inc.

Becker G (1968). Crime and Punishment: An Economic Approach. *Journal of Political Economy*. 76(2): 169-217.

Burmeister E, Dobell AR (1970). *Mathematical Theories of Economic Growth*. Collier Macmillan Publishers.

Chakraborty B, Gupta MR (2009). Human Capital, Inequality, Endogenous Growth and Education Subsidy: A Theoretical Analysis. *Research Economics*. 63(2): 77-90.

Chea CC (2015). Empirical Studies: Corruption and Economic Growth. *American Journal of Economics*. 5(2): 183-188.

Chen SH, Guo JT (2013). Progressive Taxation and Macroeconomic (In)stability with Utility-Generating Government Spending. *Journal of Economic Dynamics and Control*. 37(5): 950-205.

D'Agostino G, Dunne P, Pieroni L (2016). Government Spending, Corruption and Economic Growth. *World Development*. 84(C): 190-205.

De Hek PA (2005). On Taxation in a Two-Sector Endogenous Growth Model with Endogenous Labor Supply. *Journal of Economic Dynamics and Control*. 30(4): 655-85.

De Vaal A, Ebben W (2011). Institutions and the Relation between Corruption and Economic Growth. *Review of Development Economics*. 15(1): 108-23.

Del Monte A, Papagni E (2001). Public Expenditure, Corruption, and Economic Growth: The Case of Italy. *European Journal of Political Economy*. 17(1): 1-16.

Dzhumashev R (2014). Corruption and Growth: The Role of Governance, Public Spending, and Economic Development. *Economic Modelling*. 37(C): 201-15.

Ehrlich I, Lui FT (1999). Bureaucratic Corruption and Endogenous Economic Growth. *Journal of Political Economy*. 107(6): S270-S293.

Glaeser E, Saks R (2006). Corruption in America. *Journal of Public Economics*. 90(6-7): 1053-72.

Glomm G, Ravikumar B (1997). Productive Government Expenditures and Long-run Growth. *Journal of Economic Dynamics and Control*. 21(1): 183-204.

Gómez MA (2008). Fiscal Policy, Congestion, and Endogenous Growth. *Journal of Public Economic Theory*. 10(4), 595-622.

Gyimah-Brempong K (2002). Corruption, Economic Growth and Income Inequality in Africa. *Economics and Government*. 3(3): 183-209.

Hochman O (1981). Land Rents, Optimal Taxation and Local Fiscal Independence in an Economy with Local Public Goods. *Journal of Public Economics*. 15(1): 59-85.

Huntington SP (1968). *Political Order in Changing Societies*. Yale University Press.

Judd K (1987). A Dynamic Theory of Factor Taxation. *American Economic Review*. 77(2): 42–48.

Leff NH (1964). Economic Development Through Bureaucratic Corruption. *The American Behavioral Scientist*. 8(3): 8-14.

Lucas RE (1988). On the Mechanics of Economic Development. *Journal of Monetary Economics*. 22(1): 3-42.

Lui FT (1996). Three Aspects of Corruption. *Contemporary Economic Policy*. 14(3): 26-29.

Mankiw G, Weinzierl M (2006). Dynamic Scoring: A Back-of-the-Envelope Guide. *Journal of Public Economics*. 90(8-9): 1415–1433.

Mauro P (1995). Corruption and Growth. *Quarterly Journal of Economics*. 110(3): 681-712.

Mauro P (1998). Corruption and the Composition of Government Expenditure. *Journal of Public Economics*. 69(2): 263-79.

Mino K (1996). Analysis of a Two-Sector Model of Endogenous Growth with Capital Income Taxation. *International Economic Review*. 37(1): 227-51.

Mino K (2001). Optimal Taxation in Dynamic Economies with Increasing Returns. *Japan and the World Economy*. 13(3): 235-53.

Park H (2009). Ramsey Fiscal Policy and Endogenous Growth. *Economic Theory*. 39(3): 377-98.

Rivera-Batiz FL (2001). International Financial Liberalization, Corruption, and Economic Growth. *Review of International Economics*. 9(4): 727-37.

Rose-Ackerman S (1999). *Corruption and Government: Causes, Consequences, and Reform*. Cambridge University Press.

Sano K, Tomoda Y (2010). Optimal Public Education Policy in a Two Sector Model. *Economic Modelling*. 27(5): 991-995.

Schmitt-Grohe S, Uribe M (1997). Balanced-Budget Rules, Distortionary Taxes and Aggregate Instability. *Journal of Political Economy*. 105(5): 976–1000.

Shi SY, Temzelides T (2004). A Model of Bureaucracy and Corruption. *International Economic Review*. 45(3): 873-906.

Solow R (1956). A Contribution to the Theory of Growth. *Quarterly Journal of Economics*. 70(1): 65-94.

Stokey NL, Rebelo S (1995). Growth Effects of Flat-Rate Taxes. *Journal of Political Economy*. 103(3): 519-50.
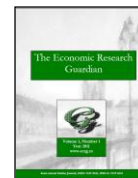
Swaleheen M, Stansel D (2007). Economic Freedom, Corruption and Growth. *Cato Journal*. 27(3): 343-358.

Teles VK (2007). Institutional Quality and Endogenous Economic Growth. *Journal of Economic Studies*. 34(1): 29-41.

Turnovsky SJ (2000). Fiscal Policy, Elastic Labor Supply, and Endogenous Growth. *Journal of Monetary Economics*. 45(1): 185-210.

Turnovsky SJ (2004). The Transitional Dynamics of Fiscal Policy: Long-Run Capital Accumulation and Growth. *Journal of Money, Credit, and Banking*. 36(5): 883-910.

Uzawa H (1961). On a Two-Sector Model of Economic Growth. *Review of Economic Studies*. 29(1): 47-70.

Uzawa H (1965). Optimal Technical Change in an Aggregative Model of Economic Growth. *International Economic Review*. 6(1): 18-31.

Wijkander H (1984). Provision of Public Goods in Congested Cities. *Journal of Public Economics*. 25(1-2): 127-141.

Zhang J (2003). Optimal Debt, Endogenous Fertility, and Human Capital Externalities in a Model with Altruistic Bequests. *Journal of Public Economics*. 87(7-8): 1825-1835.
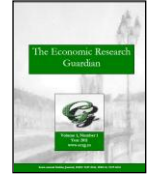
Zhang WB (1993). Woman's Labor Participation and Economic Growth - Creativity, Knowledge Utilization and Family Preference. *Economics Letters*. 42(1): 105-110.

Zhang WB (2005). *Economic Growth Theory*. Ashgate.

Zhang WB (2015). National Education and Global Economic Growth: A Synthesis of the Uzawa-Lucas Two-Sector Model and the Oniki-Uzawa Trade Model. *Journal of the Knowledge Economy*. 6(4): 905-928.

Zhang WB (2016). Impact of Education Subsidies and Taxation on Wealth and Human Capital Accumulation. *Eastern European Business and Economics Journal*. 2(3): 222-247.

Zhang WB (2017). Corruption and Growth in a Dynamic General Equilibrium Model. *European Journal of Multidisciplinary Studies*. 8(1): 256-273.

Appendix

From (4) and (6) we have

$$z \equiv \frac{r + \delta_k}{w_1} = \frac{N_i}{\overline{\beta}_i\,K_i} = \frac{N_e}{\overline{\beta}_e\,K_e},$$

(A1)

where $\overline{\beta}_q \equiv \beta_q / \alpha_q$. From (A1), (3) and (4), we have

$$r(z,\,G) = \alpha_i\,\overline{\phi}_i\,A_i\,\overline{\beta}_i^{\,\beta_i}\,G^{\theta_i}\,z^{\beta_i} - \delta_k,\quad w_1(z,\,G) = \frac{\beta_i\,\overline{\phi}_i\,A_i\,G^{\theta_i}\,z^{-\alpha_i}}{\overline{\beta}_i^{\,\alpha_i}}.$$

(A2)

Equations (3), (5), and (A1) imply

$$F_q = N_q\,f_q,\quad q = i,\,e,$$

(A3)

where $f_q(z,\,G) \equiv A_q\,G^{\theta_q} / \overline{\beta}_q^{\,\alpha_q}\,z^{\alpha_q}$. From (A3) and (6) we have

$$p = \frac{w_1}{\beta_e\,\overline{\phi}_e\,f_e}.$$

(A4)

From (10) we have

$$\overline{y}_1(z,\,G,\,a_1) = \overline{\phi}_k\,R\,a_1 + \overline{w}_1\,T_0.$$

(A5)

From (16) and (A5) we have

$$\overline{T}_1 = \sigma_1\,T_0 + \frac{\sigma_1\,\overline{\phi}_k\,R\,a_1}{\overline{w}_1},\quad \tilde{T}_1 = \frac{\eta_1\,\overline{\phi}_k\,R\,a_1}{p_1} + \frac{\overline{w}_1\,\eta_1\,T_0}{p_1}.$$

(A6)

Equations (A6) and (9) imply

$$T_1 = \overline{n}_0 - \overline{n}_1\,a_1,$$

(A7)

where

$$\overline{n}_0 = \left(1 - \sigma_1 - \frac{\overline{w}_1\,\eta_1}{p_1}\right)T_0,\quad \overline{n}_1 \equiv \left(\frac{\sigma_1}{\overline{w}_1} + \frac{\eta_1}{p_1}\right)R\,\overline{\phi}_k$$

From the definition of N_1 and (A7) we get

$$N_1 = n_0 - n_1 a_1, \tag{A8}$$

where

$$n_0 \equiv \bar{n}_0 H_1 \bar{N}_1, \quad n_1 \equiv \bar{n}_1 H_1 \bar{N}_1.$$

From (12) and (A3) we have

$$w_c = \frac{\phi_i N_i f_i + \phi_e p N_e f_e + n_c a_1 + \bar{n}_0 \phi_w H_1 w_1 \bar{N}_1}{\bar{N}_2}, \tag{A9}$$

where we also use (A7) and

$$n_c \equiv \frac{\phi_k R \bar{N}_1 - \bar{n}_1 \phi_w H_1 w_1 \bar{N}_1}{\bar{N}_2}.$$

From (A1), (A8) and (2) we have

$$\bar{\beta}_i K_i + \bar{\beta}_e K_e = \frac{n_0}{z} - \frac{n_1 a_1}{z}. \tag{A10}$$

From (20) we have

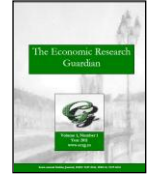$$K_i + K_e = k_1 \bar{N}_1 + k_2 \bar{N}_2. \tag{A11}$$

From (A11) and (22)

$$K_i + K_e + D = a_1 \bar{N}_1 + a_2 \bar{N}_2. \tag{A12}$$

From (A12) and (A10) we solve

$$
\begin{aligned}
K_e &= \frac{\tilde{\beta} n_0}{z} - \frac{\bar{\beta} n_1 a_1}{\bar{\beta}_i z} - a_1 \bar{\beta} \bar{N}_1 - a_2 \bar{\beta} \bar{N}_2 + \bar{\beta} D, \\
K_i &= -\frac{\tilde{\beta} n_0}{z} + a_2 \tilde{\beta} \bar{\beta}_e \bar{N}_2 + \left(\bar{\beta}_e \bar{N}_1 + \frac{n_1}{z} \right) \tilde{\beta} a_1 - \tilde{\beta} \bar{\beta}_e D,
\end{aligned}
\tag{A13}
$$

where

$$\overline{\beta} \equiv \left(\frac{\overline{\beta}_e}{\overline{\beta}_i} - 1\right)^{-1}, \quad \widetilde{\beta} \equiv \frac{\overline{\beta}}{\overline{\beta}_i}.$$

By (A1) and (A12) we have

$$N_i = \widetilde{n}_{i0} + \widetilde{n}_{i1}\, a_1 + \widetilde{n}_{i2}\, a_2 - \overline{\beta}_0\, D, \quad N_e = \widetilde{n}_{e0} + \widetilde{n}_{e1}\, a_1 + \widetilde{n}_{e2}\, a_2 + \overline{\beta}_0\, D, \tag{A14}$$

where

$$\widetilde{n}_{i0} \equiv -\overline{\beta}\, n_0, \quad \widetilde{n}_{i1} \equiv \left(\overline{\beta}_e\, z\, \overline{N}_1 + n_1\right)\overline{\beta}, \quad \widetilde{n}_{i2} \equiv \overline{\beta}\, \overline{\beta}_e\, z\, \overline{N}_2,$$
$$\widetilde{n}_{e0} \equiv \overline{\beta}_e\, \widetilde{\beta}\, n_0, \quad \widetilde{n}_{e1} \equiv -\widetilde{\beta}\, \overline{\beta}_e\, n_1 - \overline{\beta}\, \overline{\beta}_e\, z\, \overline{N}_1, \quad \widetilde{n}_{e2} \equiv -\overline{\beta}\, \overline{\beta}_e\, z\, \overline{N}_2, \quad \overline{\beta}_0 \equiv \overline{\beta}\, \overline{\beta}_e\, z,$$

By the definition of $\overline{y}_2$ we have

$$\overline{y}_2 = \widetilde{y}_0 + \widetilde{y}_1\, a_1 + \widetilde{y}_2\, a_2 + \widetilde{y}_d\, D, \tag{A15}$$

where we use (A9), (A14) and

$$\widetilde{y}_0 \equiv \overline{w}_2\, T_0 + \frac{\overline{n}_0\, \phi_w\, H_1\, w_1\, \overline{N}_1}{\overline{N}_2} + \widetilde{n}_{e0}\, \frac{\phi_e\, p\, f_e}{\overline{N}_2} + \widetilde{n}_{i0}\, \frac{\phi_i\, f_i}{\overline{N}_2}, \quad \widetilde{y}_1 \equiv \frac{n_c}{\overline{N}_2} + \frac{\widetilde{n}_{i1}\, \phi_i\, f_i}{\overline{N}_2} + \frac{\widetilde{n}_{e1}\, \phi_e\, p\, f_e}{\overline{N}_2},$$
$$\widetilde{y}_2 \equiv R + \frac{\widetilde{n}_{i2}\, \phi_i\, f_i}{\overline{N}_2} + \frac{\widetilde{n}_{e2}\, \phi_e\, p\, f_e}{\overline{N}_2}, \quad \widetilde{y}_d \equiv -\frac{\overline{\beta}_0\, \phi_i\, f_i}{\overline{N}_2} + \frac{\overline{\beta}_0\, \phi_e\, p\, f_e}{\overline{N}_2}.$$

From (22), (A3), and (A14), we have

$$\frac{\eta_1\, \overline{y}_1\, \overline{N}_1}{p_1} + \frac{\eta_2\, \overline{y}_2\, \overline{N}_2}{p_2} = f_e\, \widetilde{n}_{e0} + f_e\, \widetilde{n}_{e1}\, a_1 + f_e\, \widetilde{n}_{e2}\, a_2 + f_e\, \overline{\beta}_0\, D. \tag{A16}$$

Insert (A5) and (A15) in (A16)

$$\overline{u}_1\, a_1 + \overline{u}_2\, a_2 + \overline{u}_d\, D + \overline{u}_0 = 0, \tag{A17}$$

where

$$\overline{u}_0 \equiv \frac{\eta_2 \overline{N}_2 \tilde{y}_0}{p_2} + \frac{\overline{w}_1 \eta_1 \overline{N}_1 T_0}{p_1} - f_e \tilde{n}_{e0}, \quad \overline{u}_1 \equiv \frac{\overline{\phi}_k R \eta_1 \overline{N}_1}{p_1} + \frac{\tilde{y}_1 \eta_2 \overline{N}_2}{p_2} - \tilde{n}_{e1} f_e,$$

$$\overline{u}_2 \equiv \frac{\tilde{y}_2 \eta_2 \overline{N}_2}{p_2} - \tilde{n}_{e2} f_e, \quad \overline{u}_d \equiv \frac{\tilde{y}_d \eta_2 \overline{N}_2}{p_2} - f_e \overline{\beta}_0.$$

From (1), (18) and (19) we have

$$T_2 = \left(\frac{G}{A_p}\right)^{1/\gamma} \frac{1}{H_2 \overline{N}_2}. \tag{A18}$$

From (16) and (9) we have

$$T_2 = T_0 - \left(\frac{\eta_2}{p_2} + \frac{\sigma_2}{\overline{w}_2}\right) \overline{y}_2. \tag{A19}$$

From (A18) and (A19) we have

$$\overline{y}_2 = \Omega_0(z, G, h_1, h_2) \equiv \left[T_0 - \left(\frac{G}{A_p}\right)^{1/\gamma} \frac{1}{H_2 \overline{N}_2}\right] \left(\frac{\eta_2}{p_2} + \frac{\sigma_2}{\overline{w}_2}\right)^{-1}. \tag{A20}$$
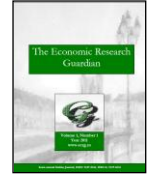
From (A20) and (A15) we have

$$D = \overline{\Omega}(z, G, h_1, h_2, a_1, a_2) \equiv \frac{\Omega_0 - \tilde{y}_0 - \tilde{y}_1 a_1 - \tilde{y}_2 a_2}{\tilde{y}_d}. \tag{A21}$$

Insert (A21) in (A17)

$$a_1 = \tilde{\Omega}(z, G, h_1, h_2, a_2) \equiv \left[\overline{u}_2 a_2 + \frac{\overline{u}_d \Omega_0 - \overline{u}_d \tilde{y}_0 - \overline{u}_d \tilde{y}_2 a_2}{\tilde{y}_d} + \overline{u}_0\right] \left(\frac{\overline{u}_d \tilde{y}_1}{\tilde{y}_d} - \overline{u}_1\right)^{-1}. \tag{A22}$$

We show now that all the variables can be expressed as functions of z, G, a_2, h_1, and h_2 as follows: r and w_1 by (A2) $\rightarrow$ f_i and f_e by (A3) $\rightarrow$ p by (A4) $\rightarrow$ $\overline{w}_1$ by definition $\rightarrow$ $\overline{w}_2$ by (11) $\rightarrow$ p_j by definition $\rightarrow a_1$ by (A22) $\rightarrow D$ by (A21) $\rightarrow \overline{y}_2$ by (A20) $\rightarrow \overline{y}_1$ by (A5) $\rightarrow \overline{y}_1$ by (A5) $\rightarrow c_j$, s_j, $\tilde{T}_j$ and $\overline{T}_j$ by (16) $\rightarrow T_1$ by (A7) $\rightarrow T_2$ by (A18) $\rightarrow N_j$ by (1) $\rightarrow K_j$ by (A13) $\rightarrow N_j$ by (A1) $\rightarrow F_j$ by (A3) $\rightarrow I_p$ by (19) $\rightarrow Y_p$ by (20). By this procedure, (17), (21), (24), and (26), we have

$$\dot{a}_1 = \Omega_1(z, G, h_1, h_2, a_2) \equiv s_1 - a_1, \tag{A23}$$

$$\dot{a}_2 = \Omega_2(z, G, h_1, h_2, a_2) \equiv s_2 - a_2, \tag{A24}$$

$$\dot{D} = \Omega_D(z, G, h_1, h_2, a_2) \equiv r D + Y_p - I_p, \tag{A25}$$

$$\dot{h}_j = \Omega_{hj}(z, G, h_1, h_2, a_2) \equiv \frac{\upsilon_{ej} \left(F_e / \overline{N}\right)^{a_{ej}} \left(h_j^{m_j} \tilde{T}_{ej}\right)^{b_{ej}}}{h_j^{\pi_{ej}}} - \delta_{hj} h_j, \quad j = 1, 2. \tag{A26}$$

We take derivatives of (A21) and (A22) with respect to time

$$\dot{D} = \frac{\partial \overline{\Omega}}{\partial z} \dot{z} + \frac{\partial \overline{\Omega}}{\partial G} \dot{G} + \frac{\partial \overline{\Omega}}{\partial h_1} \dot{h}_1 + \frac{\partial \overline{\Omega}}{\partial h_2} \dot{h}_2 + \frac{\partial \overline{\Omega}}{\partial a_2} \dot{a}_2,$$

$$\dot{a}_1 = \frac{\partial \tilde{\Omega}}{\partial z} \dot{z} + \frac{\partial \tilde{\Omega}}{\partial G} \dot{G} + \frac{\partial \tilde{\Omega}}{\partial h_1} \dot{h}_1 + \frac{\partial \tilde{\Omega}}{\partial h_2} \dot{h}_2 + \frac{\partial \tilde{\Omega}}{\partial a_2} \dot{a}_2. \tag{A27}$$

Insert (A24) and (A26) in (A27)

$$\dot{D} = \frac{\partial \overline{\Omega}}{\partial z} \dot{z} + \frac{\partial \overline{\Omega}}{\partial G} \dot{G} + M_d,$$

$$\dot{a}_1 = \frac{\partial \tilde{\Omega}}{\partial z} \dot{z} + \frac{\partial \tilde{\Omega}}{\partial G} \dot{G} + M_a, \tag{A28}$$
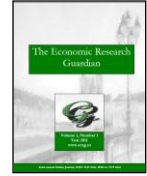
where

$$M_d(z, G, h_1, h_2, a_2) \equiv \Omega_{h1} \frac{\partial \overline{\Omega}}{\partial h_1} + \Omega_{h2} \frac{\partial \overline{\Omega}}{\partial h_2} + \Omega_2 \frac{\partial \overline{\Omega}}{\partial a_2},$$

$$M_a(z, G, h_1, h_2, a_2) \equiv \Omega_{h1} \frac{\partial \tilde{\Omega}}{\partial h_1} + \Omega_{h2} \frac{\partial \tilde{\Omega}}{\partial h_2} + \Omega_2 \frac{\partial \tilde{\Omega}}{\partial a_2}.$$

Insert (A23) and (A25) in (A28)

$$\frac{\partial \overline{\Omega}}{\partial z} \dot{z} + \frac{\partial \overline{\Omega}}{\partial G} \dot{G} = \Omega_D - M_d,$$

$$\frac{\partial \tilde{\Omega}}{\partial z} \dot{z} + \frac{\partial \tilde{\Omega}}{\partial G} \dot{G} = \Omega_1 - M_a. \tag{A29}$$

Solve linear equations (A29)

$$\dot{z} = \Omega_z\big(z,\, G,\, h_1,\, h_2,\, a_2\big),$$
$$\dot{G} = \Omega_G\big(z,\, G,\, h_1,\, h_2,\, a_2\big).$$

(A30)

We thus determined z, G, a_2, h_1, and h_2 by (A30), (A24) and (A26). We thus proved the lemma.